Habilitation Thesis: Observational Tests of The Topology of the Universe

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Abstract

General relativity matches matter-energy density to local geometry. Independent empirical measurements of both sides of the Hilbert-Einstein equations now give similar results, showing that the observable Universe is approximately flat, and giving us confidence that general relativity is a good local theory of gravity. However, the extension of general relativity which determines global geometry is not yet known, so the only feasible empirical approach is to try to measure global geometry by direct, geometrical tests. The work which I present for my habilitation thesis concerns the development of direct, geometrical tests using observational catalogues of quasars, clusters of galaxies and the cosmic microwave background.

Contents

1 Introduction

Is it possible to observe the whole Universe? Is the object studied in what is claimed to be observational cosmology really all of space or just a tiny bit of space?

In order to answer these questions, it is necessary to measure the mathematical properties of *local geometry* (such as the curvature) and *global geometry* (such as the topology), which together describe the 'shape' and size of space, under the assumption that the curvature is nearly constant everywhere in space. Since a century ago, Schwarzschild (1900), de Sitter (1917), Friedmann (1924) and Lemaître (1958) have realised that the spatial part of our Universe could correspond to a space (a 3–manifold) which may have either a non–zero curvature and/or a non–trivial topology.

The measurement of these properties (one local and the other global) from surveys obtained at telescopes of different sorts, such as the GMRT, the AAT, the VLT, XMM, MAP and Planck Surveyor, should enable us to find out if our cosmological observations are global in the sense of measuring the whole of space, or whether they simply measure a tiny fraction of the Universe: our observable sphere.

Different tests for measuring curvature or topology are dependent to differing extents on assumptions of the cosmological model adopted, and especially to assumptions on the statistics of density perturbations and the way in which these lead to galaxy formation. The tests in this habilitation are restricted to the most standard model, i.e. the 'Hot Big Bang' model, or in other words, the perturbed Friedmann–Lemaître model. The "covergence" of estimates of the curvature parameters (*local* cosmological parameters) over the last five years, via many different observational tests, covering quite different redshift ranges and making different assumptions, gives confidence that this model is a good approximation to reality.

The *physical* interpretation of the values of the local cosmological parameters, where about 25% of the matter-energy density is composed of a hypothetical "cold dark matter" particle which interacts weakly with normal matter, and about 70% consists of a negative-pressure-like parameter which is popularly called "dark energy", or more technically, "the cosmological constant" or "quintessence", remains a completely open question.

In this work, attempts at providing a theoretical interpretation of these local cosmological parameters are not made. Instead, the goal is empirical measurement of *global* cosmological parameters. If significant empirical results could be obtained for global geometry, this might provide useful clues to understanding local geometrical parameters. However, for the moment, the goal is a more limited one of simply obtaining empirical estimates.

In order to aid the non–specialist, some reminders on curvature and topology are provided in Sect. 1.1. The application of these geometrical concepts to the standard hot Big Bang model, but also some comments on extrapolations of the standard model, are provided in Sect. 1.2.

What do the observations tell us? Observational work with what may be hoped to be sufficiently deep surveys to determine the global geometry of the Universe has only just started in the last decade. My own approach has been to push this work towards real observational catalogues and to consider observationally realistic systematic effects and biases.

The following papers six papers deal with the use of (1) quasars, (2) clusters of galaxies, (3) a consequence for local cosmological parameter estimates and (2.iv), (4) the cosmic microwave background (hereafter, CMB):

(1) quasars: **Roukema (1996)**

(2) clusters of galaxies: **Roukema & Edge (1997); Roukema & Bajtlik (1999); Roukema (2000a)**

  (i) clusters as a unique object: **Roukema & Edge (1997)**

  (ii) a serendipitous, candidate global geometry based on clusters: **Roukema & Edge (1997)**

  (iii) further analysis of the candidate from optical sky surveys: **Roukema & Bajtlik (1999)**

  (iv) further analysis of the candidate using CMB data from the COBE satellite: **Roukema (2000a)**

(3) use of multiply image clusters to find high precision constraints on curvature: **Roukema & Luminet (1999)**

(4) example of a global geometry consistent with the COBE data where the size of the Universe is 10% of the horizon diameter: **Roukema (2000b)**.

This work is discussed in the context of all the various strategies for determining global geometry in Sect. 2.

Conventions, notation

Comoving coordinates are used to describe space throughout this text unless otherwise specified.

Most of the published papers use the 1990s notation for local cosmological parameters.

The present value of the matter density parameter, expressed in units of the density which would imply zero curvature if the cosmological constant is zero, is Ω_m (1990s notation Ω_0).

The present value of the dimensionless cosmological constant or "dark energy" is Ω_Λ (1990s notation λ_0).

The present value of the Hubble parameter, which sets a time scale is H_0.

1.1 Some basic geometry: curvature and topology

In the standard Friedmann–Lemaître cosmology, the model of space–time is locally based on the Hilbert–Einstein equations, where local geometry (curvature) is equated to local physical content (density) of the Universe. Such a space–time has spatial sections (i.e. hypersurfaces at constant cosmological time) which are of constant curvature.

In order to intuitively understand curvature, it is useful to use a two–dimensional analogy. An example of a flat, or curvature zero, two–dimensional space is the Euclidean plane (R^2).

Two examples of non–zero (but constant) curvature two–dimensional spaces (or surfaces) are the sphere (S^2) and the hyperboloid (H^2), which are of positive and negative curvature respectively.

These three spaces are *simply connected*, i.e. any closed loop on their surfaces can be continuously contracted to a point. This would not be the case if there was a 'handle' added to one of these surfaces, because in that case any loop circling the hole of the handle (for example) would not be contractible to a point. A space for which there exist non–contractible loops is called *multiply connected.*

An example of a flat, multiply connected space is the *flat torus* (T^2). There are three different ways to think of this space, each useful in different ways, explained further below and in Fig. 1:

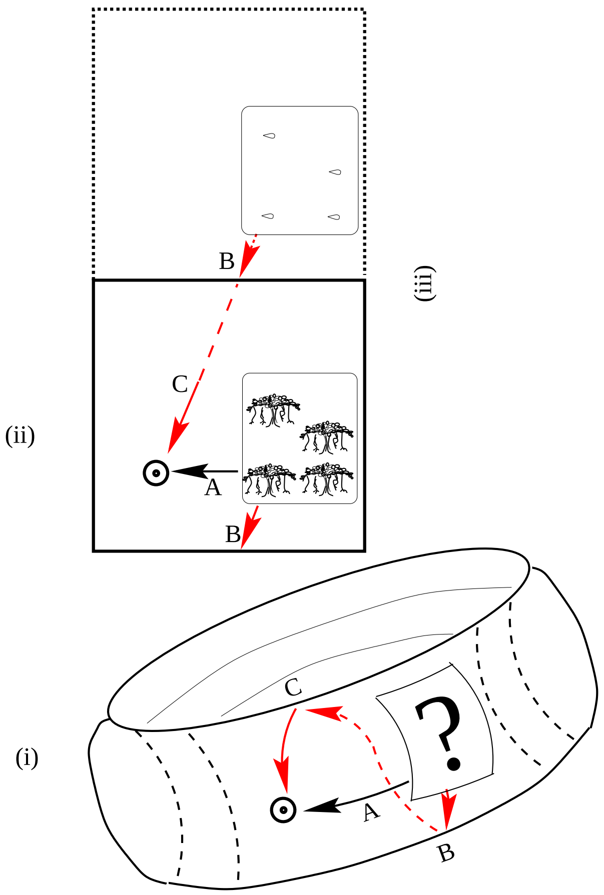

Figure 1: Schematic illustration to show different ways of thinking of a 2–torus, T^2, and how photons may come to the observer from astrophysical objects by multiple paths. The lower part of the figure shows (i): T^2 'stretched' into R^3, Euclidean 3–space (and then projected into the plane of the paper), and possible paths from the object '?' to the observer either via the short path A or via the longer path BC. The lower rectangle of the upper part of the figure shows (ii): T^2 as a rectangle with identified sides, where again two different paths to the observer, A and BC, are shown from the astrophysical object, now shown as a 'configuration' of banyan trees (to use a familiar object). The two rectangles together show part of (iii): since the path BC is longer than A, the observer in fact will see the astrophysical objects as they were a long time in the past, e.g. banyan trees would be seen as seeds (if they lived several Gigayears and were visible at cosmological distances!), and could be assumed to be present in an apparent position in another copy of the original rectangle, without introducing any error into the calculation of geodesics, time lags, etc. Version (iii) also provides the simplest way to correctly calculate the angles (perspectives) of viewing an object, which change (in general) between multiple images. For example, if a left-front-right-back orientation were added to the figure, we could say that the 'banyan trees' are viewed more or less from their 'left-hand' sides here, but that their younger versions, i.e. seeds, are viewed more or less 'from the front'.

(i) as a sort of 'doughnut' shape by inserting it in a three–dimensional Euclidean space, but retaining its flat metric (rule for deducing distances between two close points),

(ii) as a rectangle of which one physically identifies opposite sides, or

(iii) as an apparent space, i.e. as a tiling of the full Euclidean plane by multiple apparent copies of the single physical space.

The polygon (or in three dimensions, the polyhedron) of (ii) is termed the *fundamental polyhedron* (or *Dirichlet domain*). The apparent space (iii) is termed the *universal covering space*, or *covering space* for short. Representation (i) is not generally useful for analysis of observations.

One can shift between (i) and (ii) by cutting (i) the 'doughnut' shape twice and unrolling to obtain (ii), or by rolling and sticking together opposite sides of (ii) the rectangle in order to obtain (i). These operations help us to see why T^2 and R^2 are *locally* identical, i.e. both have curvature zero, since the former can be constructed from the latter by cutting a piece of the latter and pasting, but that *globally* they are different, since T^2 has a finite surface area (is 'compact'), without having any edges, but R^2 is infinite.

This can now be put in a cosmological context (imagining a two–dimensional universe), by thinking of a photon which makes several crossings of the torus T^2, i.e. of a universe. In the three ways of thinking of T^2, this can be thought of as (i) looping the torus several times, (ii) crossing the rectangle, say, from left to right several times or (iii) in the covering space (apparent space), crossing many copies of the rectangle before arriving at the observer.

Of course, this is only possible if the time needed to cross the rectangle is less than the age of the universe.

In three dimensions, the three simply connected constant curvature spaces corresponding to those listed above are the 3–D Euclidean space, R^3, the hypersphere (S^3) and the 3–hyperboloid (H^3), and the equivalent of the torus is the hypertorus (T^3), which can be obtained by identifying opposite faces of a cube. As for the two-dimensional case, R^3 and T^3 are locally identical but globally different.

There exist many other multiply connected 3-D spaces of constant curvature. These can each be represented by a fundamental polyhedron (like the rectangle for the case of T^2) embedded in the simply connected space of the same curvature (i.e. in R^3, S^3 or H^3), of which the faces of the polyhedron are identified in pairs in some way. The simply connected space is then the covering space, and can be thought of in format (iii) as above, as an apparent space which is tiled by copies of the fundamental polyhedron, just as the mosaic floor of a temple may be (in certain cases) tiled by a repeated pattern of a single tile.

If the physical Universe corresponds to a multiply connected space which is small enough, i.e. which is finite and for which photons have the time to cross the Universe several times, then the (apparent) observable Universe would be a part of the covering space and would contain several copies of the fundamental polyhedron. In other words, in apparent space, there could be multiple apparent copies of the single physical Universe.

The possibility of seeing several times across the Universe provides the basic principle of nearly all the methods capable of constraining or detecting the topology of the Universe: a single object (or a 3-D region of blackbody plasma) should be seen in different sky directions and at different distances (hence different emission epochs). These multiple images, such as the three images which, according to the observationally inspired hypothesis of **Roukema & Edge (1997)**, could be three images of a single cluster of galaxies (Coma) seen at three different redshifts, are called *topological images*.

For a thorough introduction to the subject (but prior to the recent surge in observational projects), see Lachièze-Rey & Luminet (1995). For more recent developments, see Luminet (1998) and Luminet & Roukema (1999), and workshop proceedings in Starkman (1998) and Blanlœil & Roukema (2000).

1.2 Friedmann–Lemaître–Robertson–Walker universes and their extensions

So, in order to begin to know the 'shape' of the Universe, both the curvature *and* the topology need to be known.

However, virtually all of the observational estimates of cosmological parameters have been estimates of local cosmological parameters. The curvature parameters Ω_m (present value of the matter density parameter expressed in units of the density which would imply zero curvature if the cosmological constant is zero) and Ω_Λ (present value of the dimensionless cosmological constant) and H_0 (the Hubble constant, which sets a time scale) are each defined locally at a point in space.

Estimates of the values of these parameters are now honing in rapidly, and a convergence from multiple observational methods for the three parameters is likely to signal a new phase in observational cosmology. However, as explained above, this will leave unanswered the basic question: how big is the Universe? Good estimates of the curvature parameters and of H_0 will help search for cosmic topology, and will constrain the families of spaces (3-manifolds) possible, but will be insufficient to answer the question.

Observations such as the CMB, the abundance of light elements and numerous observational statistics of collapsed objects as a function of redshift lend support to the standard FLRW or hot Big Bang model as a good approximation to the real Universe. According to this model, the age of the Universe is finite.

This condemns us to live in an observable universe which is finite, in which we are situated right at the centre, from the point of view of the universal covering space. The observable Universe can be defined as the interior of a sphere (in the covering space) of which the radius is the distance travelled by a photon that takes nearly the age of the Universe to arrive in our telescopes. The value of this radius, the horizon distance, is c/H_0 to within an order of magnitude, depending on which distance definition one uses and on the curvature parameters. This explains the common misconception according to which the value of H_0 sets the size of the Universe. The scale c/H_0 is not the size of the Universe, it is just the order of magnitude size of the *observable,* non-Copernican Universe.

In comoving coordinates (in which galaxies are, on average, stationary, where the expansion of the Universe is represented by a multiplicative factor a), and using the 'proper distance' (Weinberg, 1972, eq.14.2.21), the horizon radius is in the range $6000\mathrm{h}^{-1}$ Mpc$< R_H \lesssim 12000\mathrm{h}^{-1}$ Mpc for a generous range of curvature parameters compared to present observational constraints on the local cosmological parameters[1]. For $(\Omega_m \approx 0.3, \Omega_\Lambda \approx 0.7)$, the horizon radius is $R_H \approx 10\mathrm{h}^{-1}$ Gpc.

Note that the *observable* Universe is very non-Copernican: we are at the centre of a spherical Universe. Of course, the underlying model implies that the complete *covering space* is (maybe) much larger: finite for positive curvature, infinite for non-positive curvature, and in neither case does the covering space have a centre.

Note also that the 2-sphere S^2 does not have a centre which is part of S^2. The centre of a 2-sphere embedded in R^3 exists in R^3, but is not part of the 2-sphere. S^2 can be very easily defined as a mathematical object independently of R^3. The embedding in R^3 is certainly a useful mathematical tool, and an aid to intuition, but is not at all necessary. So, if S^2 corresponds to a physical object, this does not imply that R^3 has physical meaning, nor that the 'R^3-centre' of S^2 has any physical meaning. The exactly corresponding arguments apply to S^3 relative to R^4.

If Robertson and Walker's implicit hypothesis that the topology of the Universe is trivial were correct (the hypothesis according to which, for example, the 3-torus T^3 is a priori excluded), then, since the observations seem to indicate that the Universe is either negatively curved (hyperbolic) or flat, not only would the covering space be infinite, but the Universe itself would be! This would imply that the fraction of the Universe which is observable would be zero, since the observable Universe is finite. It would also imply (for a constant average density, the standard assumption) that the mass of the Universe is infinite.

This may or may not be correct. Atoms have finite masses, as do photons, trees, people, planets and galaxies. *If the Universe is a physical object, then extrapolation from better known physical objects would suggest that it should also have a finite mass.*

The alternative possibility, that the Universe is a spiritual object, requires an implicit acceptance of dualism, which on length scales studied by the cognitive sciences, usually in the range $\sim 10^{-6} - 10^{1}$m, is judged by many to be self-contradictory, unnecessary, and/or confused, though debate still continues (e.g. Searle (1980) in favour of dualism; Hauser (1993); Miller (1997) against dualism). This possibility will not be further discussed in this habilitation.

Both theoretical and observational methods can be used to examine the hypothesis of trivial topology.

Many theoretical cosmologists and physicists work on extensions to the standard model, to epochs preceding that during which the cosmic microwave background black-body radiation was emitted (e.g. see the early universe, topological defect and superstring cosmology papers in Bharadwaj & Kar 2000). Inflation (an accelerated expansion of the Universe at an early epoch, e.g. when the age of the Universe was $\sim 10^{-33}s$) and other theoretical ideas regarding the 'early' Universe don't invalidate the standard Big Bang model as a good approximation for post-recombination observations (i.e. probably all observations so far), even if some now include 'no Big Bang' boundary conditions at the quantum epoch $t \sim 10^{-43}s$. On the contrary, they extrapolate from the standard model.

Among these various scenarios, some treat the Universe as having infinite volume, some as finite, and many do not state either way.

If we consider one of the early Universe models in which the volume is infinite or else, say, the Universe is globally a hypersphere with radius 10^3 times that of the horizon, and if we assume that the topology of the Universe is trivial, then a more or less serious question of credibility arises: is the extrapolation from the observable Universe to the entire Universe 10^3 times or infinitely many times bigger justified? Is an extrapolation from an 'infinitesimal' (i.e. zero) fraction to the whole justified?

Whether these questions lie in the domain of physics or of the philosophy of science will not be dealt with further here, except to remark that for the sake of precision, it would be best to make it clear in literature for the non-specialist when one is studying the 'observable Universe' or the 'local Universe', and not leave the term 'Universe' without an appropriate qualifying adjective.

It is clear that if the topology is assumed to be trivial, then the measured values of local parameters such as Ω_m and H_0 would be 'local' in more than one sense of the word: local as a physical quantity, and local since the values are averaged over an 'infinitesimal' fraction or, say, a ten billionth of the total volume of the Universe.

2 Dropping the simple-connectedness hypothesis

The hypothesis that the Universe is simply connected is ... just a hypothesis.

If this hypothesis is dropped, then the whole Universe may well be *smaller* than the 'observable Universe'! The latter would then form a part of the universal covering space, and would constitute the 'apparent Universe' containing many copies of the entire physical Universe. Multiple connectedness does not necessarily imply that multiple copies would be visible (one or all dimensions might be bigger than the horizon diameter), but certainly implies this as a physical possibility.

[1] $h \equiv H_0/(100\mathrm{km\ s}^{-1}\ \mathrm{Mpc}^{-1})$.

As mentioned above, awareness that measurement of topology would be required in order to characterise the geometry of space has been around for at least a century (Schwarzschild 1900), and has been discussed by several of the symbols of modern cosmology (de Sitter 1917; Friedmann 1924; Lemaître 1958). Although measuring curvature, essentially via estimates of the density parameter, Ω_m, and the cosmological constant, Ω_Λ, has sustained much more attention and observational analyses than measurement of topology, some discussion of the latter both theoretically and in relation to the status of continually growing observational catalogues of extragalactic objects was made in the 1970's and 1980's, in particular by Ellis, Sokoloff and Schvartsman, Zel'dovich, Fang and Sato, Gott and Fagundes [see Lachièze-Rey & Luminet (1995) for a detailed reference list, also, e.g. Narlikar & Seshadri (1985); Barrow, Juszkiewicz & Sonoda (1985)].

Since the release of data from the COBE satellite, several papers were quickly published to make statements about spatial topology with respect to the COBE data (Stevens et al. 1993; Sokolov 1993; Starobinsky 1993; Fang 1993; Jing & Fang 1994). The publication of a major review paper (Lachièze-Rey & Luminet 1995) further prompted interest in the subject, so that there are now several dozen researchers in Europe, North America, Brazil, China, Japan and India actively working on various observational methods for trying to measure the topology of the Universe.

See Luminet & Roukema (1999, Section 5) for a detailed discussion of the recently developed observational methods, apart from new work which is cited below. For earlier work, which showed by various methods that the size of the physical Universe should be at least a few $100h^{-1}$ Mpc, see Lachièze-Rey & Luminet (1995).

Most of the methods depend either directly or indirectly on multiple topological imaging of either collapsed astrophysical objects or of photon-emitting regions of plasma.

Other methods are the statistical incompatibility between observable topological defects and observable cosmic topology (Uzan & Peter 1997; Uzan 1998a,b), and the suggestion of Roukema (2003) which postulates a physical and geometrical link between the apparent $L \approx 130h^{-1}$ Mpc feature in large scale structure and global topology.

The direct methods are those for which photons are expected to travel across the Universe in different directions from a single object or plasma region and arrive at a single observer. They may leave the object or plasma region at different cosmological times.

The indirect methods suppose that regions which are nearby to one another have correlated physical properties, so that although an object or plasma region is not strictly speaking multiply imaged, close by regions are approximately multiply imaged. This approach is subject to the validity of the assumptions regarding correlations over 'close' distances.

2.1 Three-dimensional methods (collapsed objects)

Researchers in France, Brazil and Poland (Fagundes & Wichoski 1987; Fagundes 1996, 1998; Lehoucq, Luminet & Lachièze–Rey 1996; **Roukema 1996**; Roukema & Blanloeil 1998; **Roukema & Bajtlik 1999**; **Roukema & Luminet 1999**; Gomero et al. 1999a; Lehoucq et al. 1999; Uzan, Lehoucq & Luminet 1999; Fagundes & Gausmann 1997, 1999a,b; Gomero, Rebouças & Teixeira 2000, 1999c) work principally on direct three-dimensional methods, i.e. study various statistical techniques which analyse the spatial positions of all known astrophysical objects at large distances inside of the observational sphere. Just as for traditional observational estimates of the local cosmological parameters Ω_m, H_0 and Ω_Λ, the idealised methods have to be adapted in practice to cope with the fact that we observe objects in the past and with astronomical selection effects. Different classes of objects have different constraints on their evolution with cosmological time, are seen to different distances and are observed in a combination of wide shallow surveys and narrow deep surveys. The result is that the search for cosmic topology — or claimed proofs of simple connectedness below a given length scale — are just as difficult as the attempts to measure the curvature parameters and the Hubble constant, efforts which have taken more than half a century in order to start coming close to convergent results.

2.1.1 (1) quasars: Roukema (1996)

One way to find a very weak signal in a population such as quasars which are likely to evolve strongly over cosmological time scales can be seen schematically in Fig. 1. This is the search for rare local isometries **(Roukema 1996)**.

Although the properties of *individual* quasars may have changed completely between the high redshift and low redshift images, the relative three-dimensional spatial positions of a configuration of quasars should remain approximately constant in comoving coordinates, since comoving coordinates are by definition approximately constant over time for a given physical object, apart from small scale movements due to peculiar velocities.

Even if such isometries are rare, use of a large enough catalogue may be enough to detect enough isometries to generate testable 3-manifold candidates, whose predictions of multiple topological imaging can be tested by other means.

This way a weak signal hidden in a large catalogue can potentially be detected.

The example analysis of an observational catalogue of quasars in **(Roukema 1996)** revealed two isometries, each an isometry between two quintuplets of quasars. The calculation was made under the simplifying assumption, reasonable at the time, that $(\Omega_m = 1.0, \Omega_\Lambda = 0.0)$. Followup to this work would be similar calculations using a range of local cosmological parameters close to $(\Omega_m = 0.3, \Omega_\Lambda = 0.7)$.

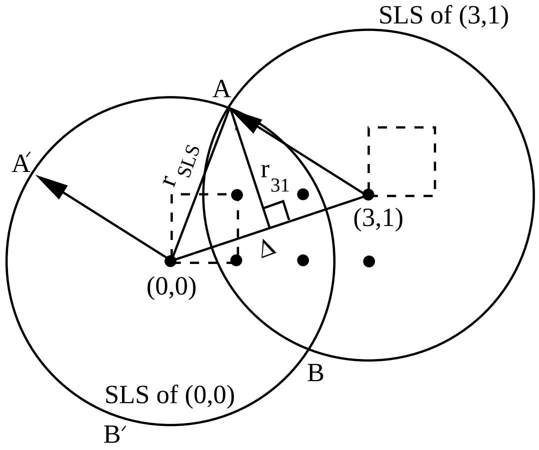

Figure 2: Illustration of 'identified circles principle', shown with one dimension less, i.e. with two 1-spheres (circles) that intersect in a 0-sphere (pair of points), instead of two 2-spheres that intersect in a 1-sphere (circle). The two 1-spheres, or surfaces of last scattering (SLS), of radius $r_{\rm SLS}$, are centred at the observer at $(0,0)$ and at a copy of the observer at $(i=3, j=1)$ in the covering space, where i and j represent numbers of copies of the fundamental polyhedron (rectangle in 2-D, rectangular prism in 3-D, shown as a dashed curve). Their intersection consists of the two points A and B (in 3-D this intersection is a full circle). The two observers looking at A and B from 'opposite' sides are equivalent to one observer, at $(0,0)$, looking at A and B in one direction, and at A' and B' in some other direction, defined by the tiling pattern of the fundamental polyhedron and coordinate location of the observer's copy. In 3-D, the two identified circles for a single observer defined by copy (i,j) are the extensions of the 0-sphere $\{A, B\}$ and the 0-sphere $\{A', B'\}$ into 1-spheres (circles). The radii of the identified circles are r_{31}, and the two circles are separated by the observer-observer copy distance, labelled Δ in the figure. Black dots are used to show some other possible positions of a copy of the observer, each of which defines a distinct pair of identified circles.

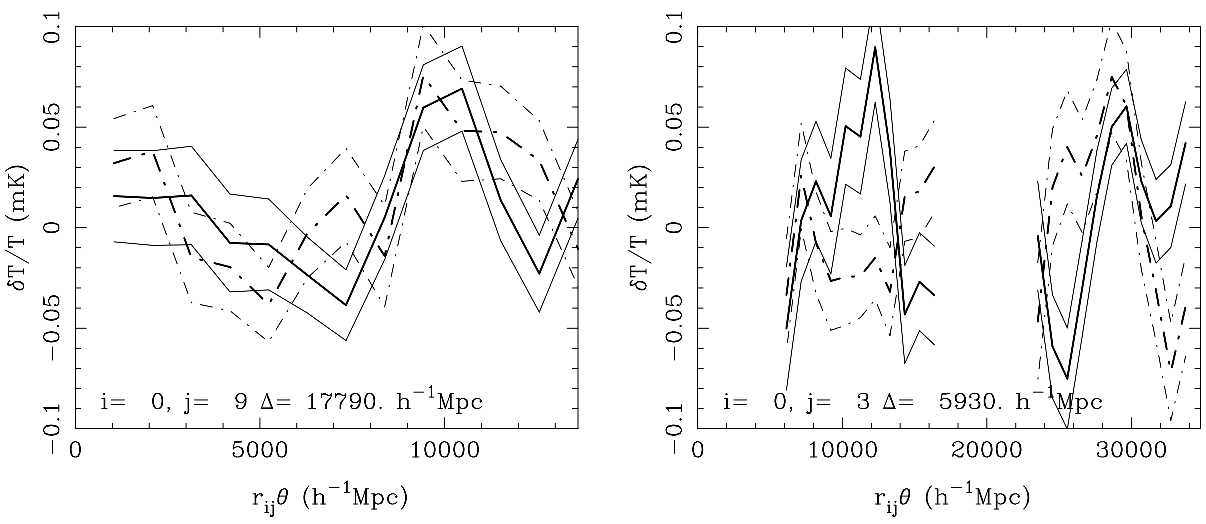

Figure 3: Two examples of would-be identified circles shown as temperature fluctuations $\delta T/T$ in the cosmic microwave background as observed by the COBE satellite. The upper panel shows $\delta T/T$ around a pair of circles implied by a model found to be consistent with the four-year COBE data, the lower panel shows $\delta T/T$ around a pair of circles for a model found inconsistent with the full data set. The circles do not cover 360°; this is due to exclusion of galactic latitudes with $|b^{\rm II}| < 20°$. Thick lines and thin lines indicate $\delta T/T$ and $\delta T/T \pm \Delta(\delta T/T)$ (uncertainties) respectively, assuming that 30% of the total signal is due to the integrated Sachs-Wolfe effect, added to the noise in quadrature. For a correct model, the $\delta T/T$ values around matched circles should be equal within the statistical limits of the uncertainties, assuming that the values are primarily due to the naïve Sachs-Wolfe effect. The horizontal axes are labelled in lengths $r_{ij}\theta$ around the circles of radii r_{ij}, for $(\Omega_m = 0.3, \Omega_\Lambda = 1-\Omega_m)$. The two models are '2-tori', i.e. $T^2 \times R$, of size one-tenth of a horizon diameter, but oriented differently in the two cases. The labels (i,j) indicate which observer-observer copy pair defines the circle pair (see Fig. 2). Each circle lies in a plane halfway between the observer and her topological image at $i(2R_H/10)\mathbf{e_x} + j(2R_H/10)\mathbf{e_y}$ and its matching circle is at $(-i,-j)$, where $\mathbf{e_x}$ and $\mathbf{e_y}$ are unit vectors in the two short directions of $T^2 \times R$. The distance between a circle and its match is indicated here by Δ.

2.1.2 (2) clusters of galaxies: Roukema & Edge (1997); Roukema & Bajtlik (1999); Roukema (2000a)

(i) clusters as a unique object: Roukema & Edge (1997)

The main point of **Roukema & Edge (1997)** was to point out that since massive clusters of galaxies are the largest and most recent objects to have virialised (become dynamically relaxed), they provide a source of very special, and in some sense unique, objects. Since one of the key observational difficulties in detecting cosmic topology is distinguishing true multiple images from physically distinct objects which just happen to have statistically common physical properties, the existence of any sort of "label" of uniqueness could offer crucial help. This point is discussed in the first part of the paper.

(ii) a serendipitous, candidate global geometry based on clusters: Roukema & Edge (1997)

To illustrate the uniqueness of massive clusters of galaxies, we prepared a short, non-systematic, list of a few of highly luminous and well-known clusters. We noticed that of these seven clusters, three (RX J1347.5, CL 09104 and Coma) happen to form a geometrical configuration which would be just that expected in the most commonly studied case of a multiply connected Universe: the case of an equal-sided torus (either a 2-torus, $T^2 \times R$, or a 3-torus, T^3).

While this is most likely a coincidence, we considered this as the basis for deriving an example candidate for the global geometry of the Universe which can be relatively easily falsified by observations.

Some initial examination of observational constraints is presented in **Roukema & Edge (1997)**, particularly for the T^3 case, but because the observational uncertainties are high, especially at redshifts about $z \approx 0.4$, the constraints are weak. Although some differences in observational parameters are discouraging for this particular topological lensing hypothesis, none are strong enough to definitively falsify it.

(iii) further analysis of the candidate from optical sky surveys: Roukema & Bajtlik (1999)

In this paper, we particularly focussed on the $T^2 \times R$ interpretation of the candidate topological lensing hypothesis in order to illustrate a method of refuting certain topological lensing hypotheses via optical data. Moreover, this analysis demonstrates that a parameter generally difficult or impossible to measure for most celestial objects – the velocity tangential to the line-of-sight, i.e. along the plane of the sky – can potentially be measured for galaxies in a cluster where two images of the cluster lie at nearly identical redshifts.

Since the two high redshift images occur at nearly identical redshifts in the case of RX J1347.5, CL 09104 and Coma, one axis of their sky projections should be in common to both. Hence, optical images of the sky regions containing these clusters should show galaxies at nearly identical positions along the common (projected) axis, and the small differences in positions should imply tangential velocities, along the plane of the sky.

If the hypothesis is false, the attempted matching of galaxies between the two images should either fail, or should yield physically unrealistic tangential velocities.

Surprisingly, the results yielded a solution rather than a contradiction to the hypothesis.

Some control tests were made, based on deliberately arbitrary topology hypotheses, of which a few yielded similar solutions, and many yielded physically unrealistic solutions (excessively high tangential velocities). This is clearly not enough to strengthen the hypothesis.

Better photometric data, and spectroscopic data, from a medium-sized telescope (2-4m) would be needed to try to refute the hypothesis using this method. (Observing time has been applied for, but so far with no time awarded.)

(iv) further analysis of the candidate using CMB data from the COBE satellite: Roukema (2000a)

The identified circles principle (Sect. 2.2.1) was used to try to refute the RX J1347.5-CL 09104-Coma identity hypothesis based on the four-year COBE data. Given certain simplifying assumptions, such as assuming that the uncertainties stated by the COBE team include all sources of error and that no further systematic errors are present, the hypothesis is rejected at the 94% confidence level. This is discouraging for the hypothesis, but not of high enough significance to definitively falsify it.

2.1.3 (3) use of multiply imaged clusters to find high precision constraints on curvature: Roukema & Luminet (1999)

This paper showed that detection of topological lensing at "moderate" redshifts ($z \lesssim 3$), with 5 or so multiple images, would provide purely geometric constraints on the *local* cosmological parameters, constraining Ω_m to least 1% precision and Ω_Λ to at least 10% precision.

Moreover, it is shown that the potential precision in estimating these local parameters, by using multiple topological images up to this redshift range in future surveys over the next half-century, could reach about 10^{-6} to 10^{-7}.

2.1.4 Related work on three-dimensional methods

In a population composed of good standard candles and negligible selection effects, the 'cosmic crystallography' method of Lehoucq et al. (1996) (or its variations) could be applied. By plotting a histogram of pair separations (in the covering space) of the objects (i.e. an unnormalised two-point correlation function), sharp 'spikes' should occur at distances corresponding to the sizes of the vectors representing the isometries between the copies of the fundamental polyhedron (*generators).* The original version of cosmic crystallography is valid only for flat spaces. However, Uzan et al. (1999) extrapolated the search for local isometries [discussed and calculated for quintuplets in **Roukema (1996)**] to the case of isometries of pairs, and defined a single statistic based on the number of 'isometric' pairs. This 'collecting-correlated-pair' statistic should have a high value in the presence of multiple topological imaging in a population composed of good standard candles, whether space is hyperbolic, flat or spherical, but only for the correct values of the curvature parameters Ω_m and Ω_Λ.

Most of the three-dimensional methods avoid having to make an a priori hypothesis regarding the precise 3-manifold (space), its size and orientation. Since there are infinitely many 3-manifolds possible, this is a considerable advantage for hypothesis testing. However, if carried out to sufficient precision and generality to guarantee a detection, in the case that an observational data set is homogeneous and deep enough and the topology of the Universe really is non-trivial, the methods generally require a lot of computing power, generally in CPU rather than in disk space.

Various ideas to improve the speed of the calculations are suggested by some of these authors. Alternatively, if some observations can be used to suggest candidate 3-manifolds (spaces), then the methods can be used on different, independent observational data sets to attempt to refute the suggested candidates relatively rapidly.

2.2 Two-dimensional methods (cosmic microwave background)

Both direct and indirect approaches have been suggested for using CMB measurements, those of COBE and future measurements by MAP and Planck Surveyor.

2.2.1 The direct approach: the identified circles principle

The direct method results from an elegant geometrical result discovered by a North American group (Cornish, Spergel & Starkman 1996, 1998). This is known as the identified circles principle.

The microwave backgrounds seen by two observers in the *universal covering space* separated by a comoving distance R lying in the range $0 < R < 2R_H$ consist of two spheres in the covering space, which intersect in a circle in the covering space. If the two observers are multiple topological images of a single observer, then what is a single circle in the covering space as seen by two observers is equivalent to two circles seen in different directions by a single copy of the observer (see Fig. 2).

This implies that for a single observer, and for locally isotropic radition on the surface of last scattering, the temperature fluctuations seen around a circle centred at one position in a CMB sky map should be identical to those seen around another circle (in a certain direction), apart from measurement uncertainty. The positions and radii of the circles are not random, they are determined by the shape, size and orientation of the fundamental polyhedron, or equivalently, by the set of generators.

Cornish et al. (1998) hope to use the MAP satellite data to apply this principle in a generic search for the topology of the Universe. It has been shown that the principle can be applied to four-year data from the COBE satellite despite its poor resolution and poor signal-to-noise ratio, either to try to refute a given topology candidate motivated by three-dimensional data **(Roukema 2000a)** or to show that a flat '2-torus' ($T^2 \times R$) model a tenth of the horizon diameter can easily be found which is consistent with the COBE data (see Fig. 3 here, or **Roukema 2000b** for details).

For the identified circles principle to be applied in a general way, i.e. to search for the correct 3-manifold rather than to test a specific hypothesis, the computing power required would again be very high, as for the three-dimensional methods. Improvement in the speed of calculation will be required for the application to MAP and Planck Surveyor data.

Another direct method, which has been tested to some degree via simulations, and which should in principle be derivable from the identified circles principle, is that of searching for patterns of 'spots' in the CMB (Levin et al. 1999).

2.2.2 (4) example of a global geometry consistent with the COBE data where the size of the Universe is 10% of the horizon diameter: Roukema (2000b).

As explained above, it was shown in **Roukema (2000a)** that the RX J1347.5-CL 09104-Coma identity hypothesis was weakly rejected by using the identified circles principle in a direct analysis of the four-year COBE data.

In **Roukema (2000b)**, it was shown that it is quite easy to find an FLRW universe model whose size is 10% of the horizon diameter and is clearly consistent (rejected at only 60% significance for the full set of circles) with the COBE data. One example of identified circles in this model is shown in the left-hand panel of Fig 3.

Moreover, it was shown that this is not simply because the COBE data is too poor quality. Even with the inclusion of a strong noise component qualitatively representing the integrated Sachs-Wolfe contribution to temperature fluctuations, a model was found which is rejected at the 99.8% significance level (full set of circles). One example of identified circles in this strongly rejected model is shown in the right-hand panel of Fig 3.

These results invalidate the *conclusions* or *statements* in several examples of the indirect approach, which claim that a universe whose size is 10% of the horizon diameter is inconsistent with COBE data, though of course they do not invalidate the *calculations* themselves, which are certainly valuable, as long as they are interpreted with caution.

2.2.3 The indirect approach: use of perturbation statistics assumptions

Many researchers (Stevens et al. 1993; Sokolov 1993; Starobinsky 1993; Fang 1993; Jing & Fang 1994; de Oliveira Costa & Smoot 1995; de Oliveira-Costa et al. 1996; Levin, Scannapieco & Silk 1998) have tried to use the indirect approach, i.e. via introducing assumptions on the density perturbation spectrum (or equivalently the correlation function of density perturbations) and making statements regarding ensembles of possible universes, as opposed to direct observational refutation. Most of these authors made simulations of the perturbations in order to obtain statements of statistical significance. Various subsets of flat 3-manifolds were tested and it was suggested that flat 3-manifolds up to 40% of the horizon diameter were inconsistent with the COBE data. As mentioned above, direct tests applying the identified circles principle show that a more conservative constraint would have to be around 10% of the horizon diameter.

A Canadian based group (Bond, Pogosyan & Souradeep 1998, 2000a,b) has tested individual hyperbolic candidates applying the perturbation statistics approach to COBE data. Since Fourier power spectra are, strictly speaking, incorrect in hyperbolic space, and since eigenmodes are difficult to calculate in compact hyperbolic spaces, these authors used correlation functions instead. Although these authors use some simulations in their figures, the perturbation statistics approach is applied by them without relying on simulations. This bypasses potential errors and numerical limitations which could be introduced by the transitions from perturbation statistics to simulations, and from simulations to final statistical statements (though it does not avoid the original assumptions).

Inoue (1999), Aurich (1999) and Cornish & Spergel (1999) calculated eigenmodes in compact hyperbolic spaces in order to apply the perturbation simulational approach. They showed that the C_l (spherical harmonic) statistic for COBE four-year data was consistent with their models, taking into account the fact that for low density universe models, i.e. for $\Omega_m < 1$, the gravitational redshifts between the observer and the surface of last scattering, known as the integrated Sachs-Wolfe effect (or the Rees-Sciama effect) makes refutation of 3-manifold candidates using CMB data more difficult than if the Universe were flat and the cosmological constant were zero.

Applications of the perturbation statistics approach (with or without simulations) to testing multiply connected models have the property that they depend on the assumptions regarding density fluctuation statistics. The latter are observationally supported by most (but not all) analyses of COBE data on large scales, *provided that* the Universe is *assumed* to be simply connected. However, this is not a valid assumption if the Universe is multiply connected. This internal contradiction is sometimes ignored in interpretations of the indirect approach.

For more discussion on this question, see Section 1.2 of **Roukema (2000a)**.

3 Summary and Prospects

Is it possible to show by observations that the global Universe is observable? And that observational cosmology is something more than just extragalactic astrophysics?

For a recent summary of observational results, see table 2 of Luminet & Roukema (1999). The analyses using different observational data sets and different methods have so far answered these questions with 'No' on scales which are much smaller than the horizon. All the observations point to the Universe being simply connected up to a scale of $\sim 1000\mathrm{h}^{-1}$ Mpc, i.e. about a twentieth of the horizon diameter.

At the $\sim$ $1000\mathrm{h}^{-1}$ Mpc scale and larger, definitive answers have not yet been obtained. On the contrary, some candidate 3-manifolds consistent with several observational data sets have been suggested by Roukema & Edge **(Roukema & Edge 1997; Roukema & Bajtlik 1999; Roukema 2000a)**, **Roukema (2000b)** and Bond, Pogosyan & Souradeep (1998, 2000a,b).

Specific testing of these large 3-manifolds might show that one of these makes correct predictions of three-dimensional positions (celestial coordinates and redshift) of multiple topological images of previously known objects.

In parallel, developments of all the techniques described above are being actively pursued by the different groups cited above, using computer simulations and different classes of observations.

Strong hopes are put in the MAP satellite whose initial data may have been released for analysis (Feb 2002) by the time this habilitation is formally submitted, but the better resolution and the ability to measure polarisation information in the CMB by the Planck Surveyor might be needed to extract the signal from the noise.

Using three-dimensional methods, numerous surveys such as the 2dF and SDSS galaxy and quasar surveys which are presently underway might be sufficient to obtain significant results.

Theory may be slow in catching up. Quantum cosmology studies regarding the evolution of topology during the quantum epoch are nowhere near making predictions for the present-day topology of space, though some interesting work has begun (e.g. Dowker & Surya 1998).

Not only would a significant measurement of cosmic topology show that the Universe is spatially finite in at least one direction, but it would add topological 'lensing' to the tool presently finding a great variety of useful applications: gravitational lensing. The two are similar in that the geometry of the Universe generates multiple images in both cases. They differ in that the former uses the whole Universe as a 'lens', does not magnify the image and generates images at (in general) widely differing redshifts and angles, all of which contrast with the latter.

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