Inhomogeneous cosmology in an anisotropic Universe

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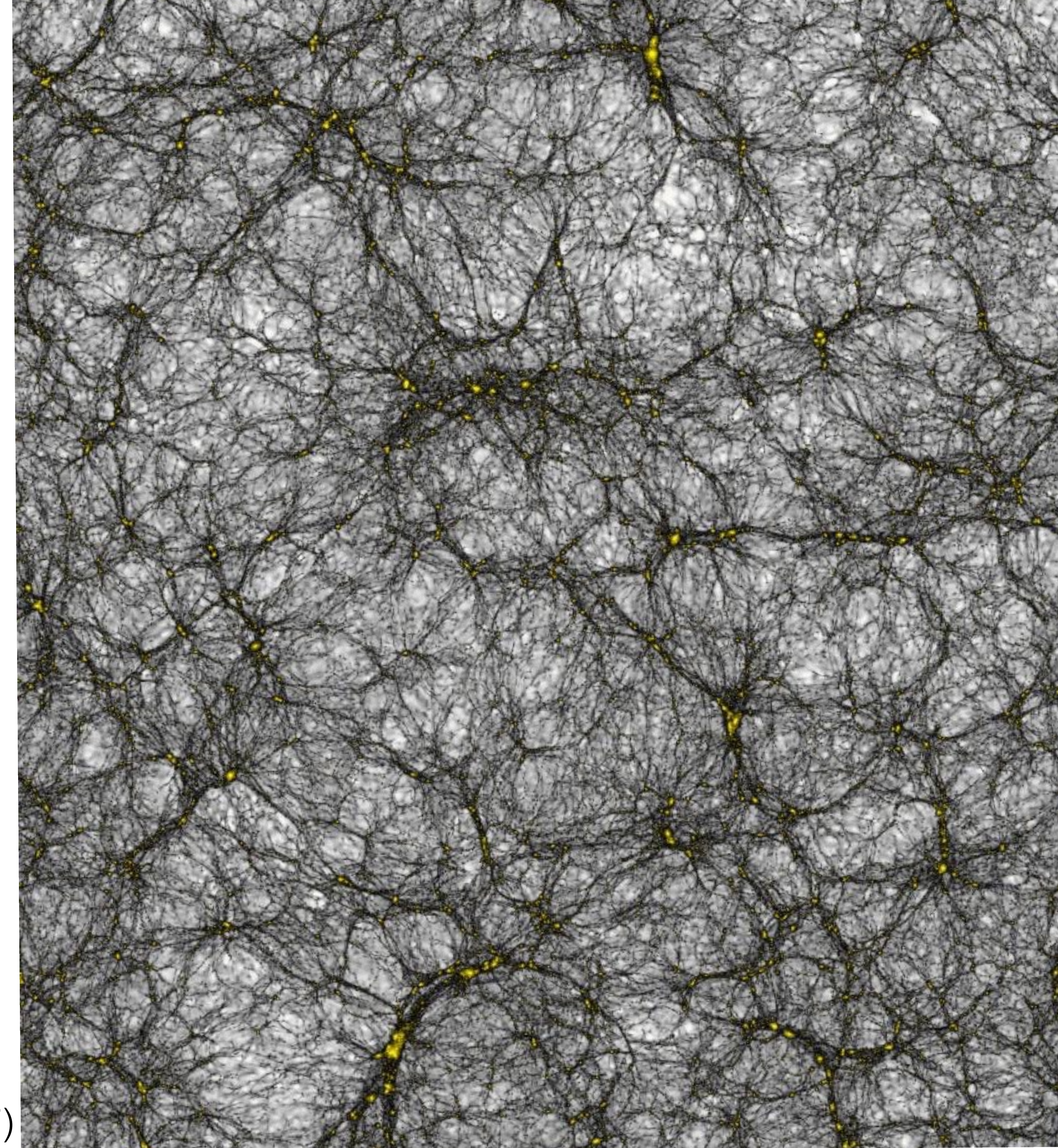
Why?

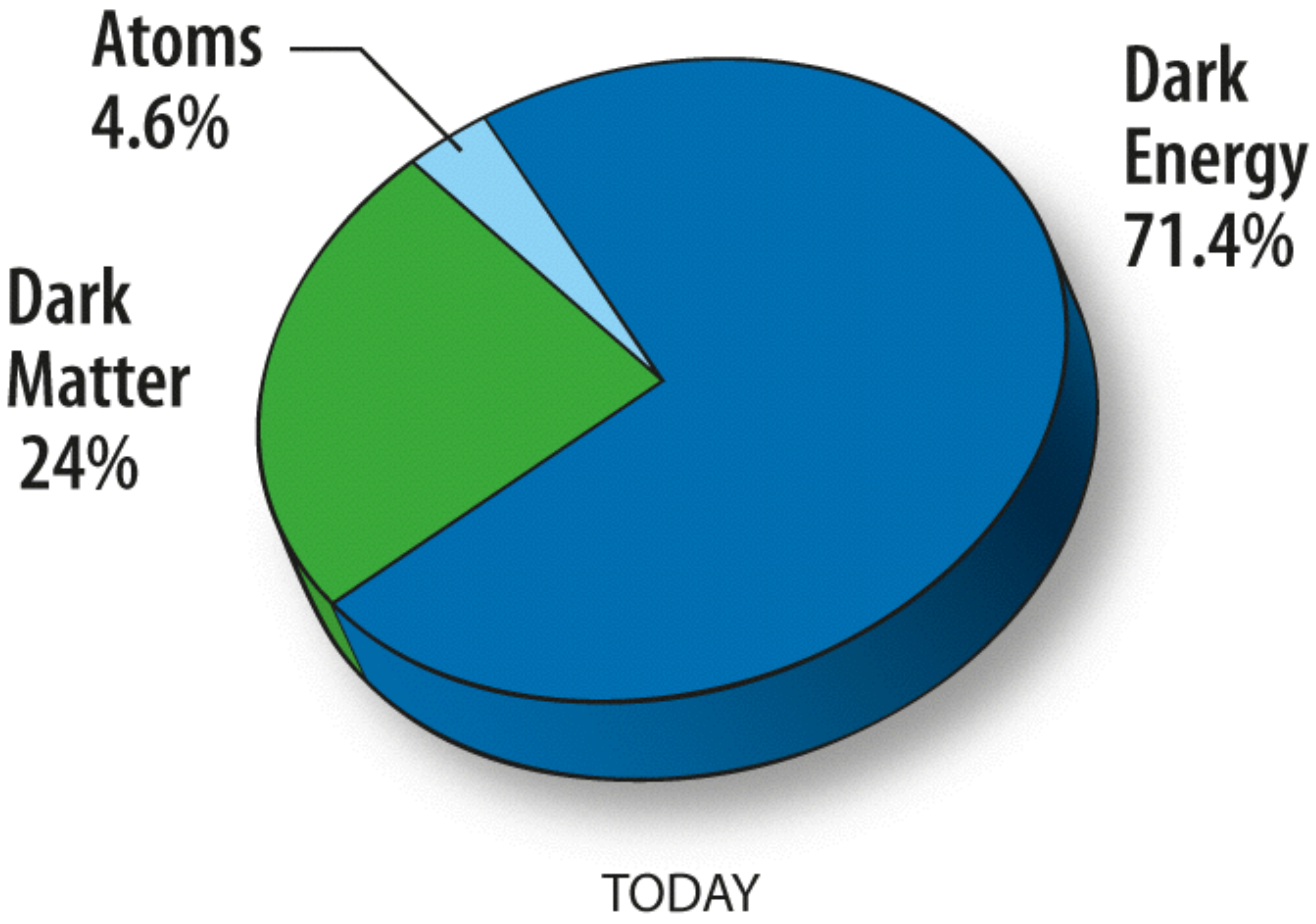
- Precision of upcoming cosmological surveys
    - Euclid, SKA, LSST
- N-body sims increasing resolution
- Must also ensure our computational models are accurate



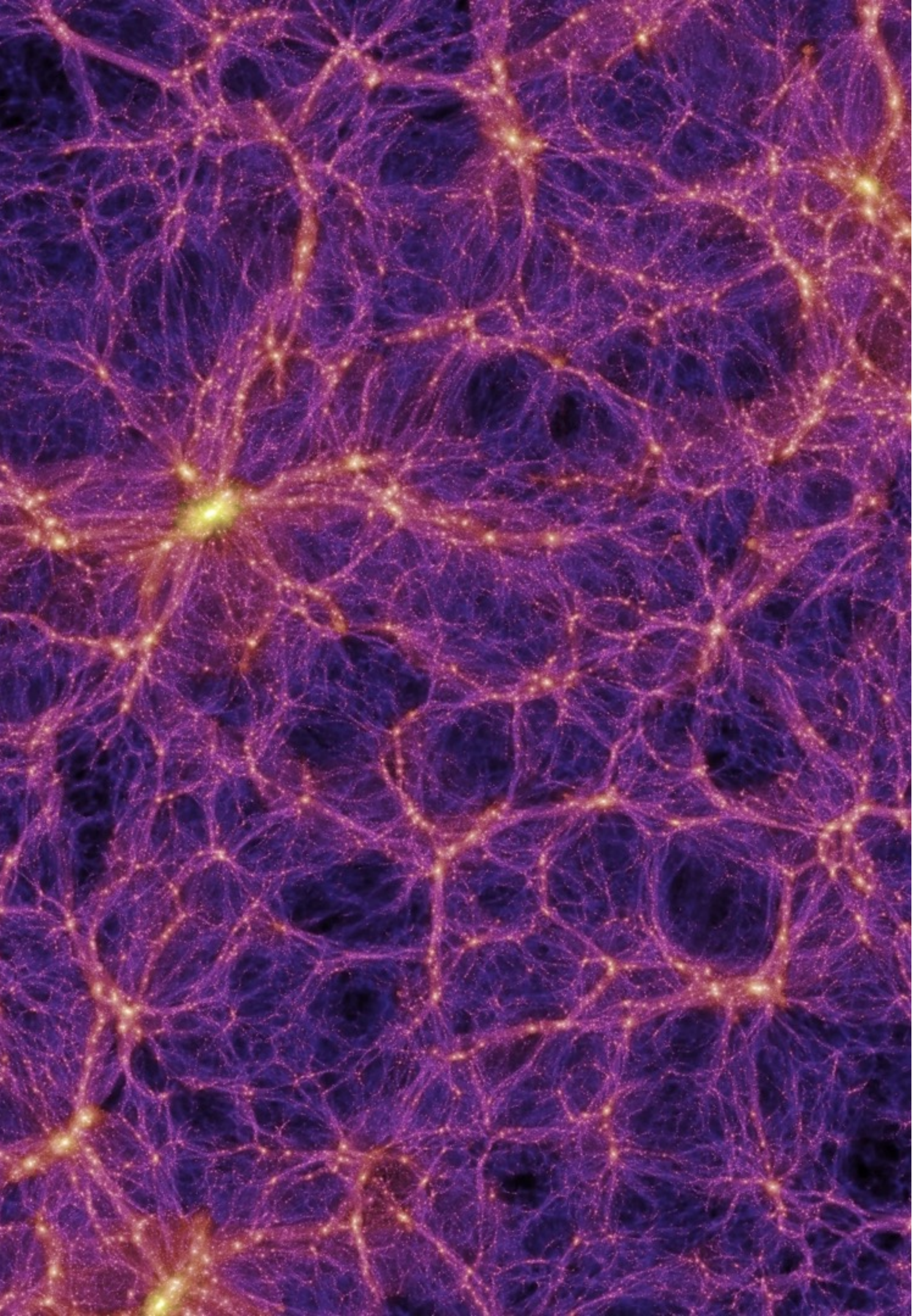
Potter et al. (2017)

The standard model

- ΛCold Dark Matter (ΛCDM)
- Successful in explaining many cosmological observations
- Based on assumptions of homogeneity and isotropy
  - flat FLRW model
- Recent tensions in H_0



Hinshaw et. al (2013)

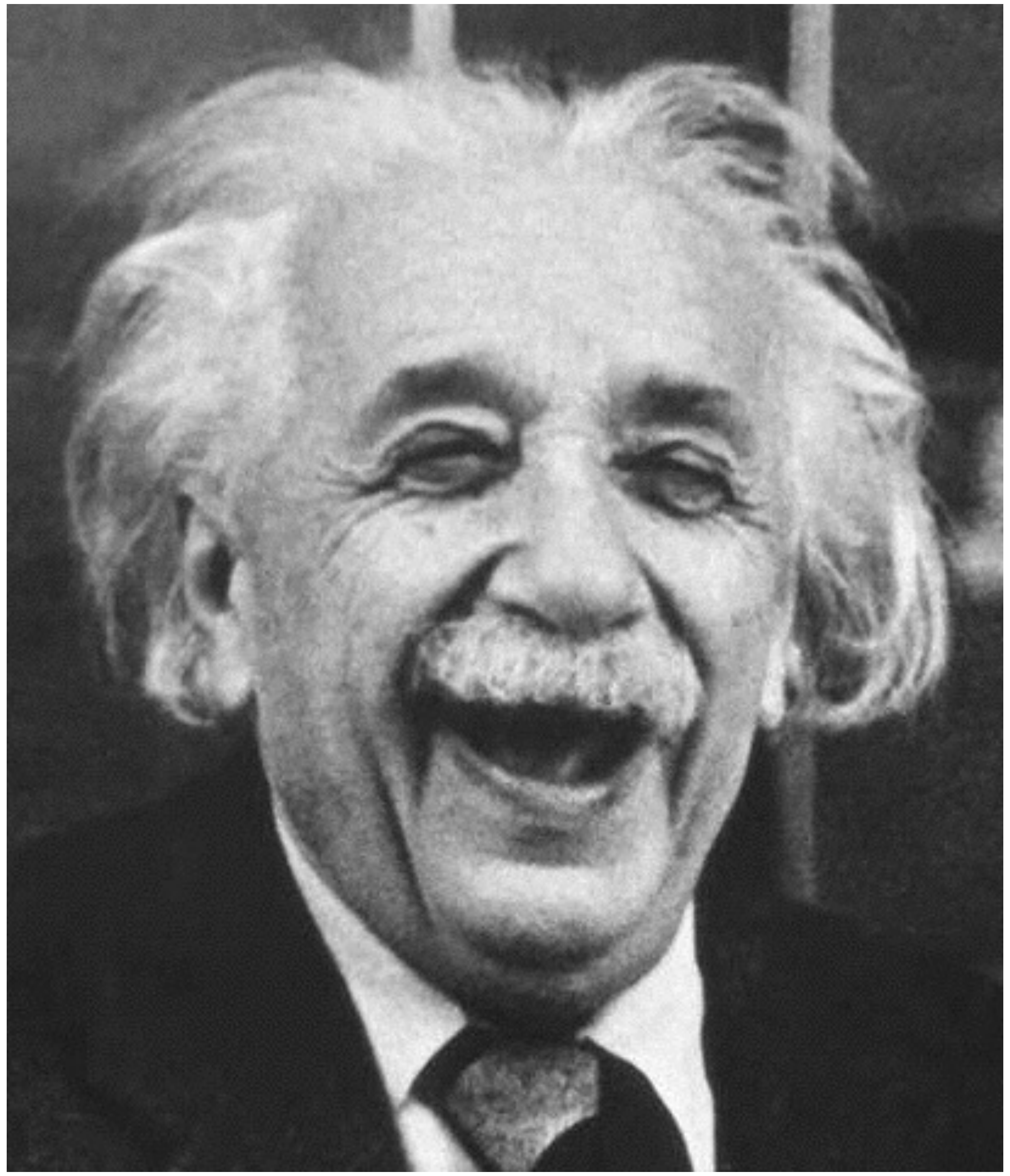


Springel et. al (2005)

Current models

- Assume homogeneously-expanding FLRW background spacetime
- Add matter on top and evolve with N-body methods
- Newtonian gravity
- Matter and spacetime **cannot** interact

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$



- 10 field equations
- Matter & spacetime intimately linked
- Our universe is "lumpy" on small scales
- "Lumpy" matter implies "lumpy" spacetime

Spacetime is **not** homogeneous

…so why should it expand homogeneously?

- Curvature affects geodesics
- Light propagation
- Observations made well below homogeneity scale $\sim 80h^{-1}\mathrm{Mpc}$

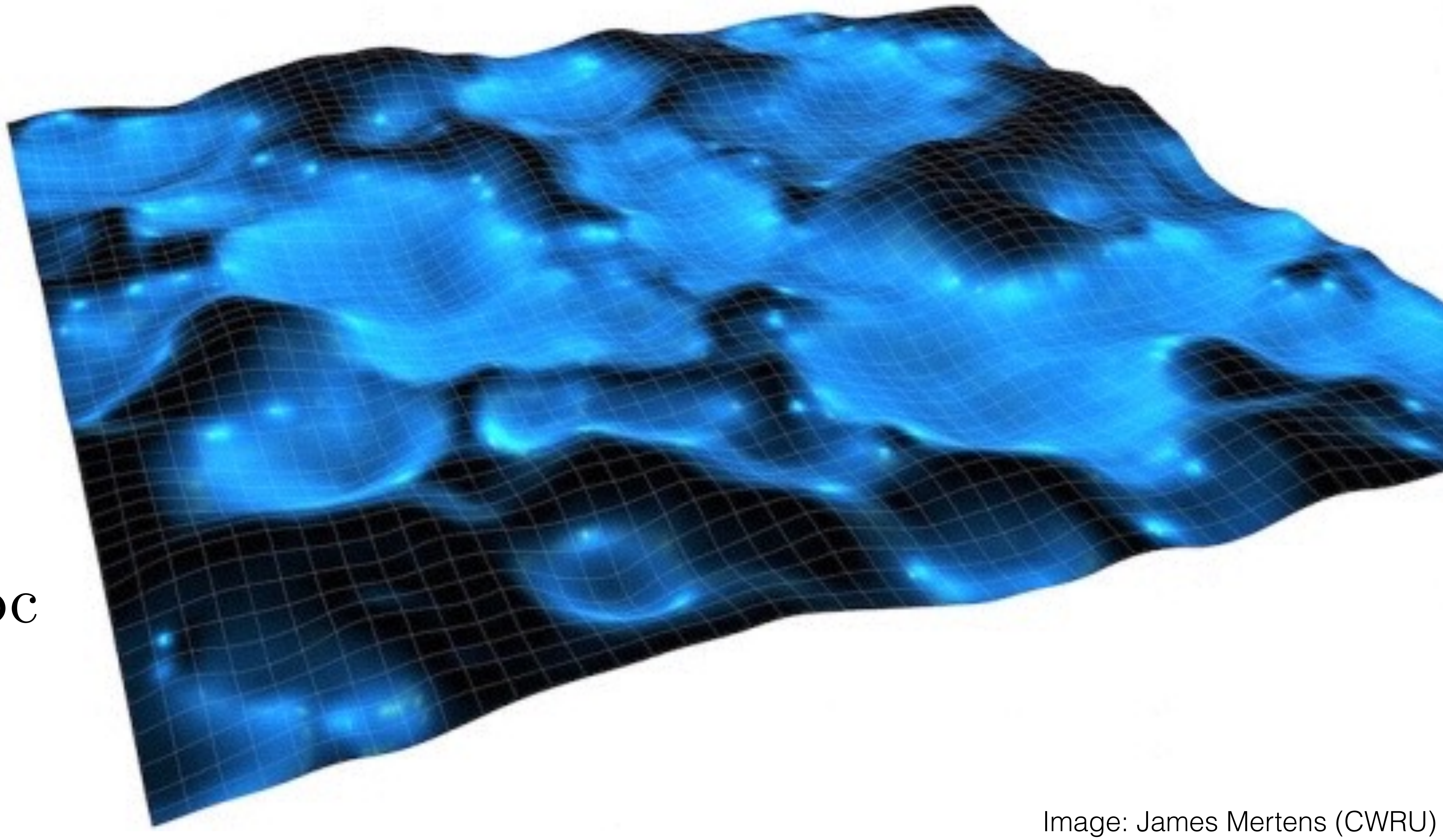
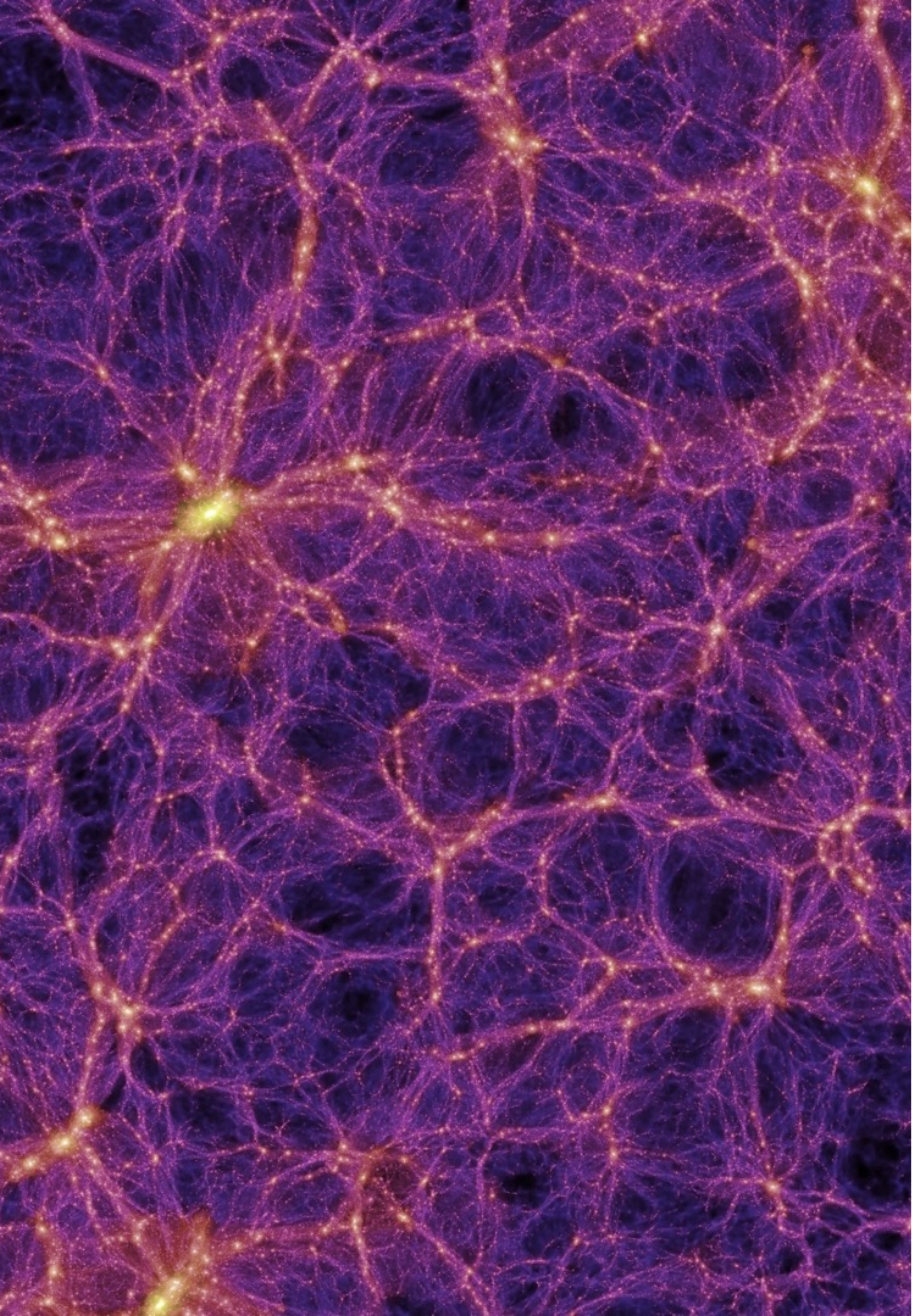


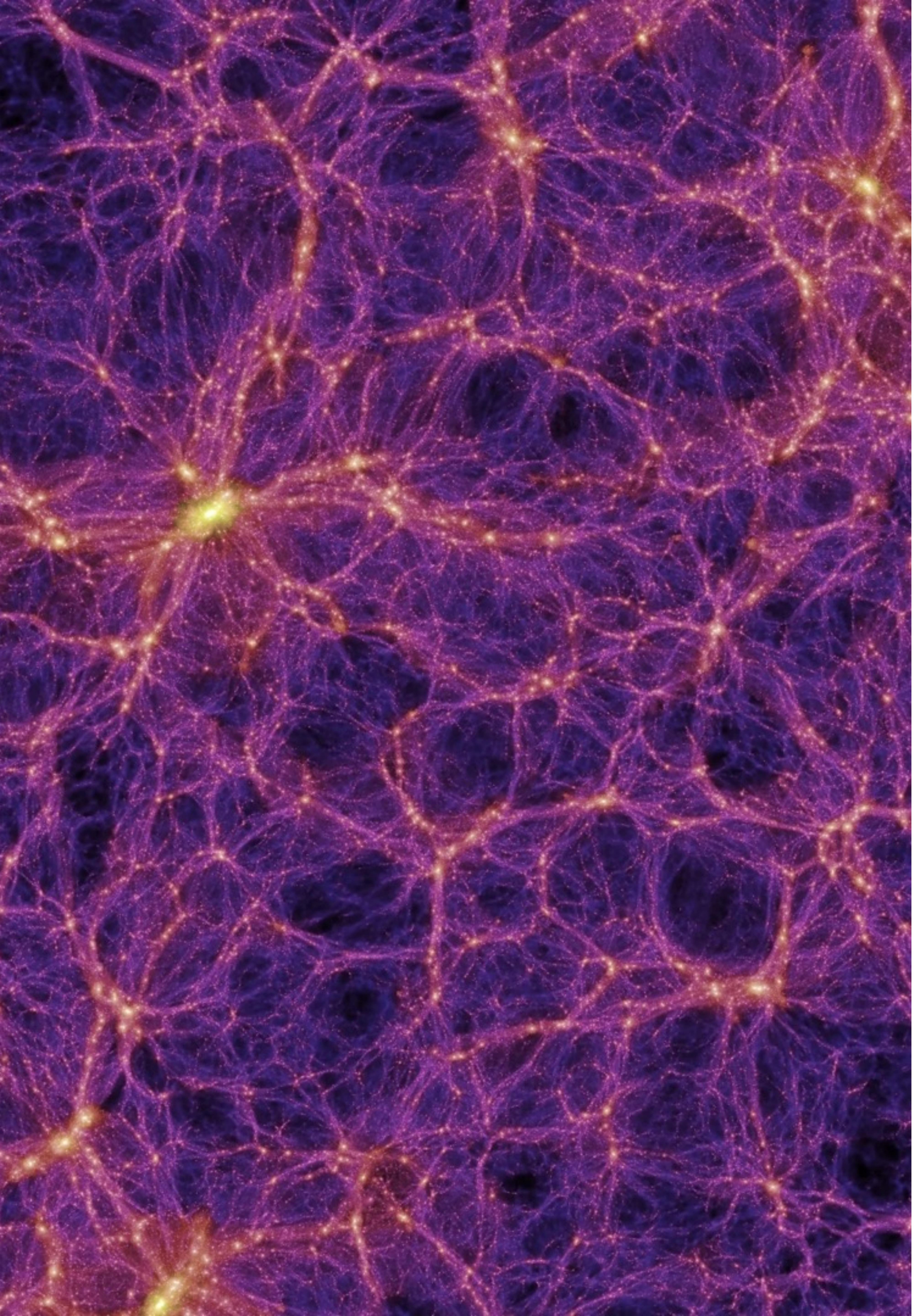
Image: James Mertens (CWRU)



Springel et. al (2005)

The "dream" model

- No assumptions of homogeneity or isotropy
- Einstein's general relativity in full
- Matter and spacetime **can** interact dynamically
- Large scale, very high resolution
- Matter evolved with N-body methods

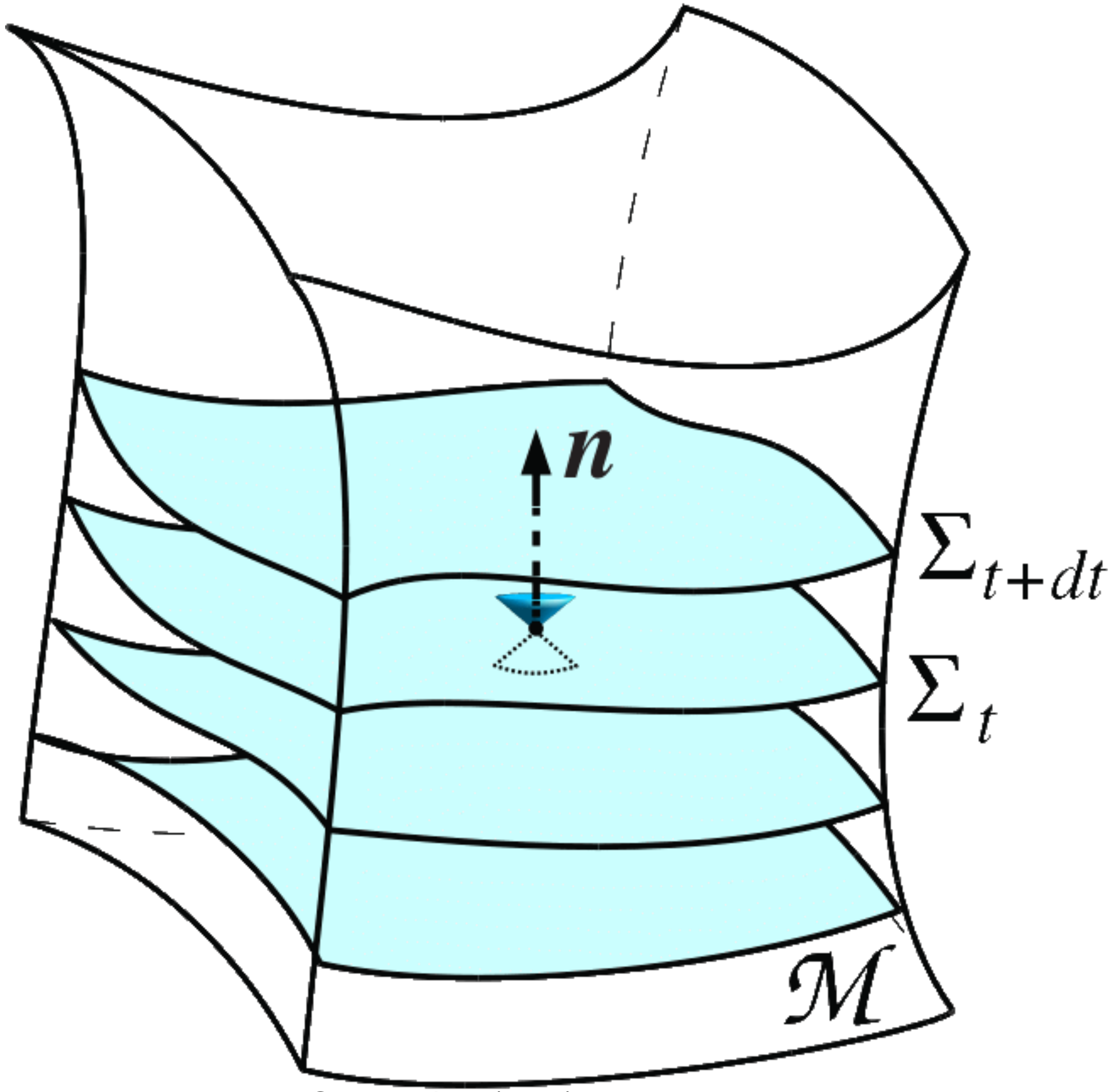


Springel et. al (2005)

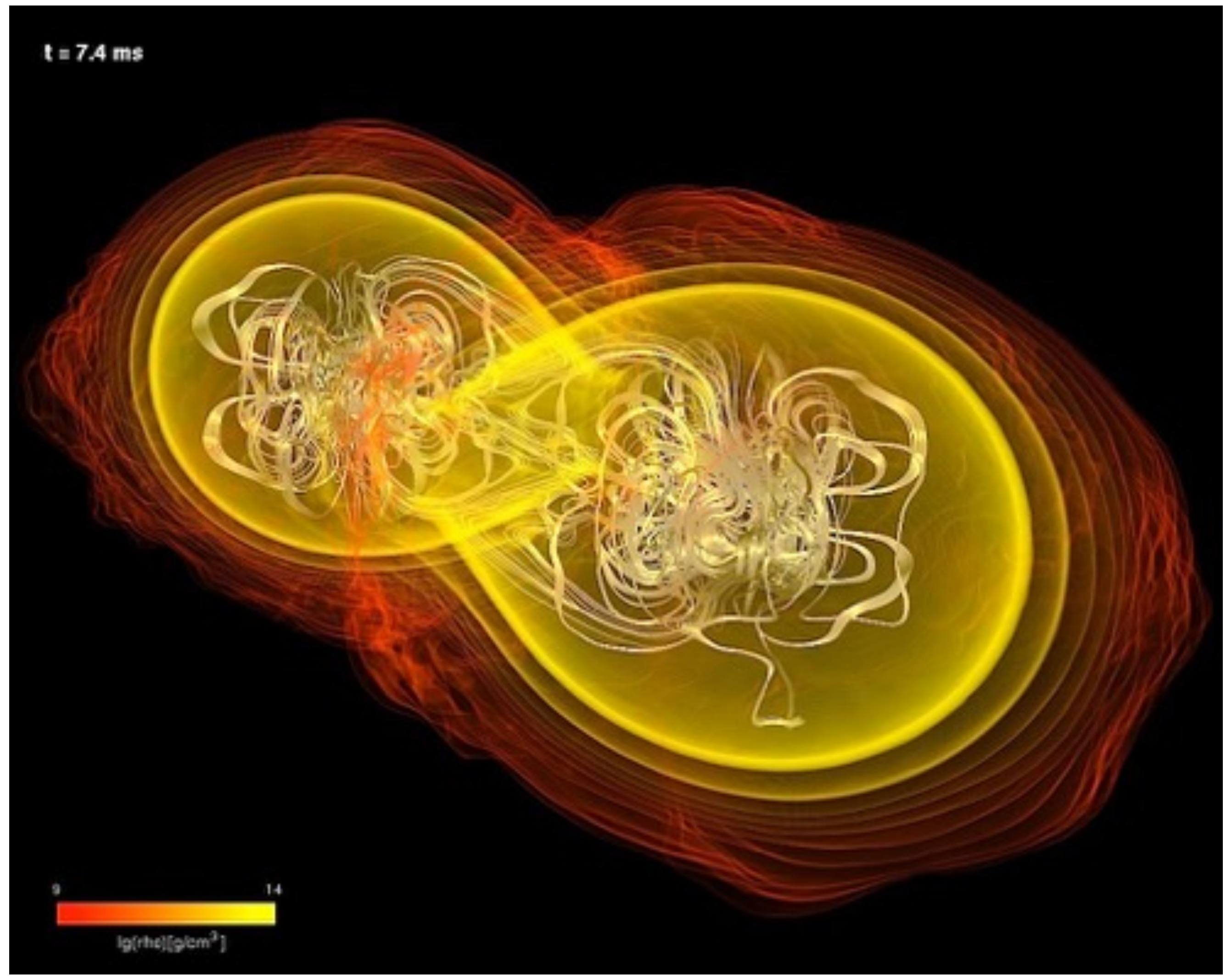
Our model

- No assumptions of homogeneity or isotropy
- Einstein's general relativity in full
- Matter and spacetime **can** interact dynamically
- Large scale, ~~very high resolution~~
- ~~Matter evolved with N-body methods~~

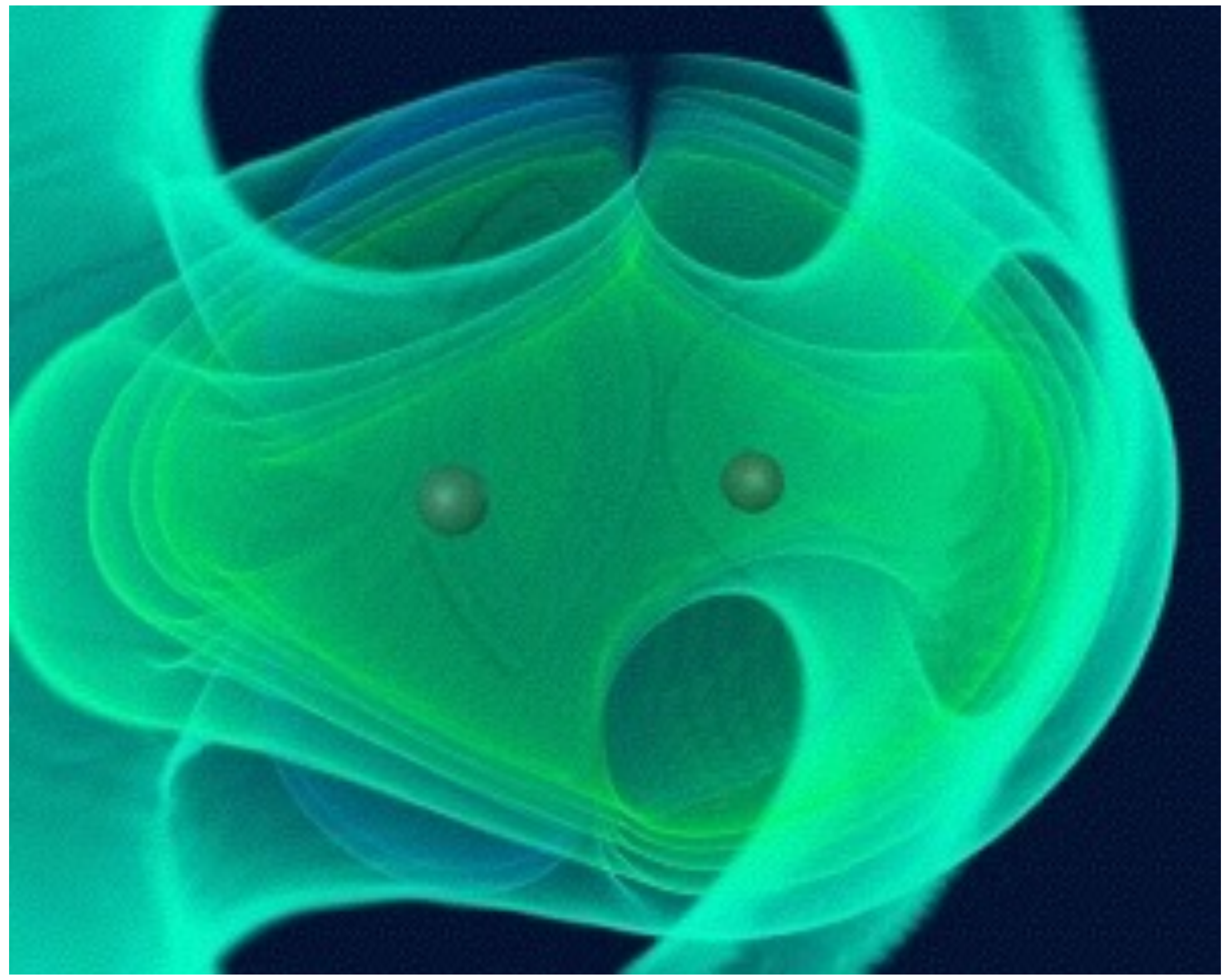
Numerical relativity



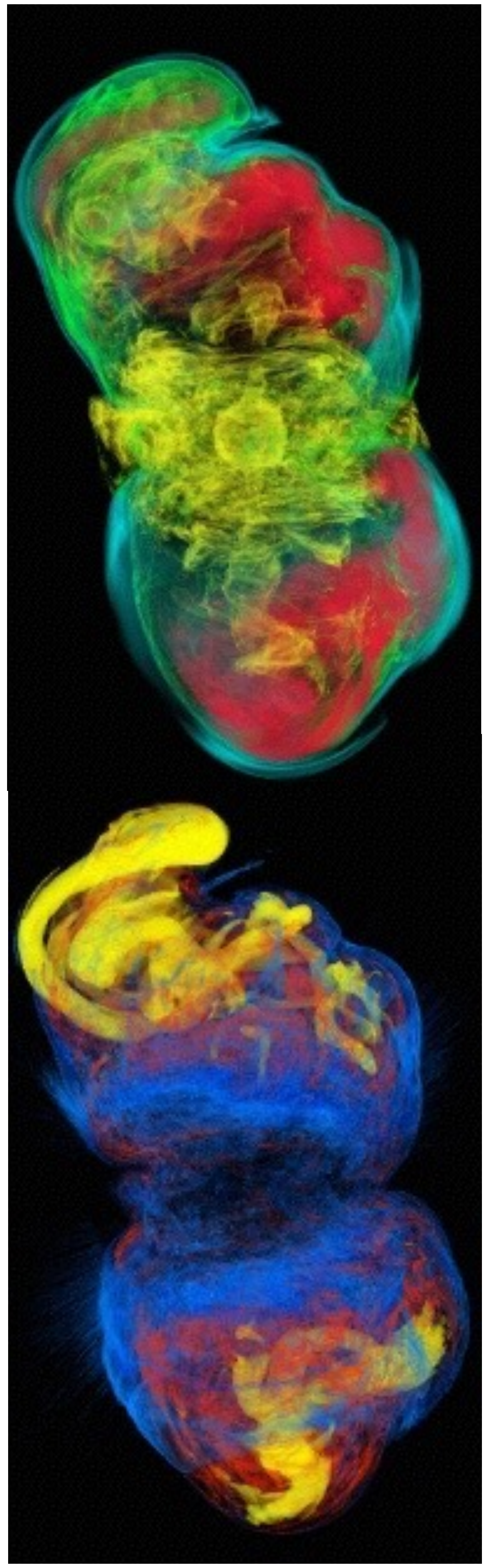
Gourgoulhon (2007)



Giacomazzo et. al (2011)



Rezzolla et. al (2010)



Moesta et. al (2014)

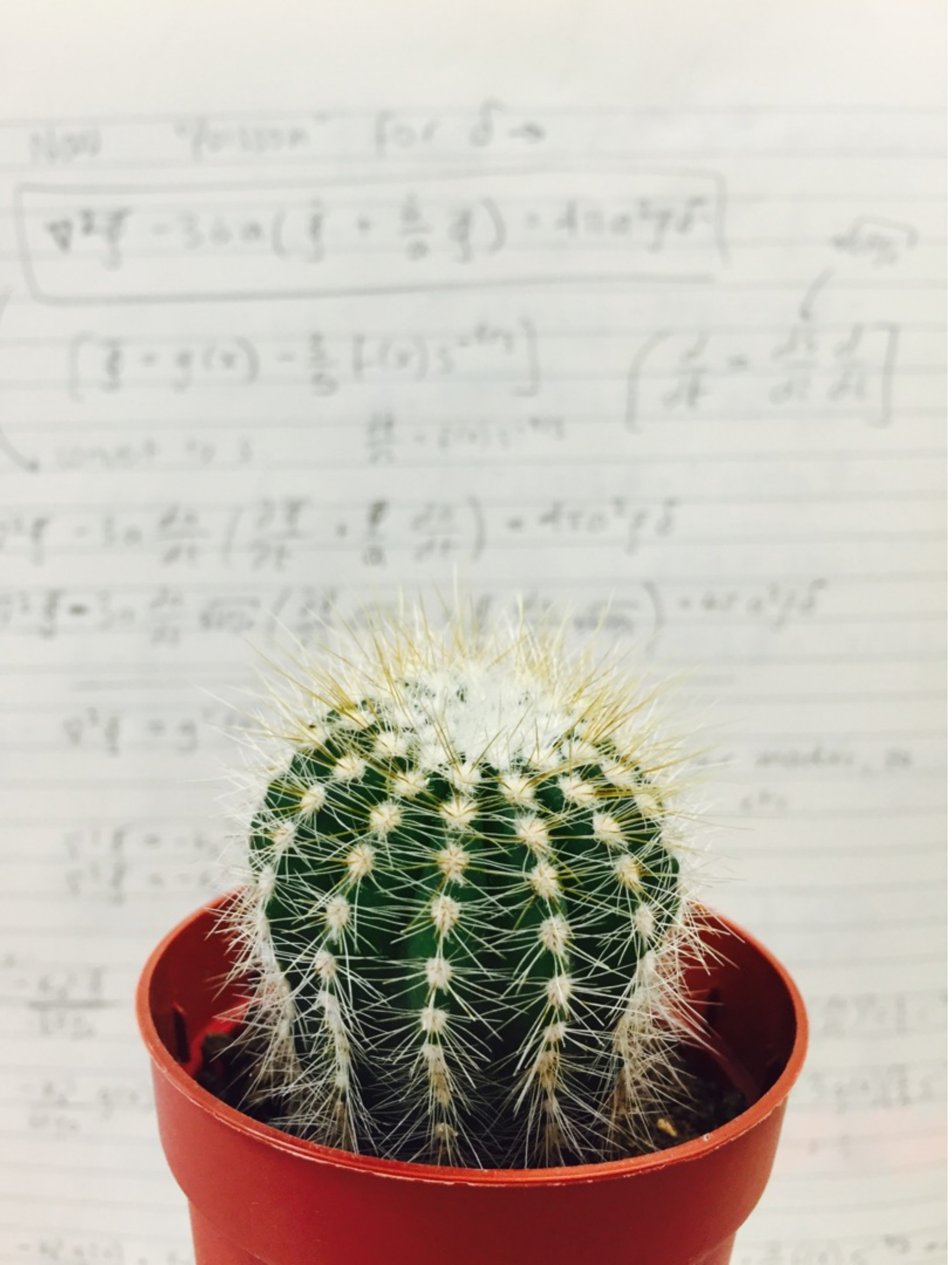
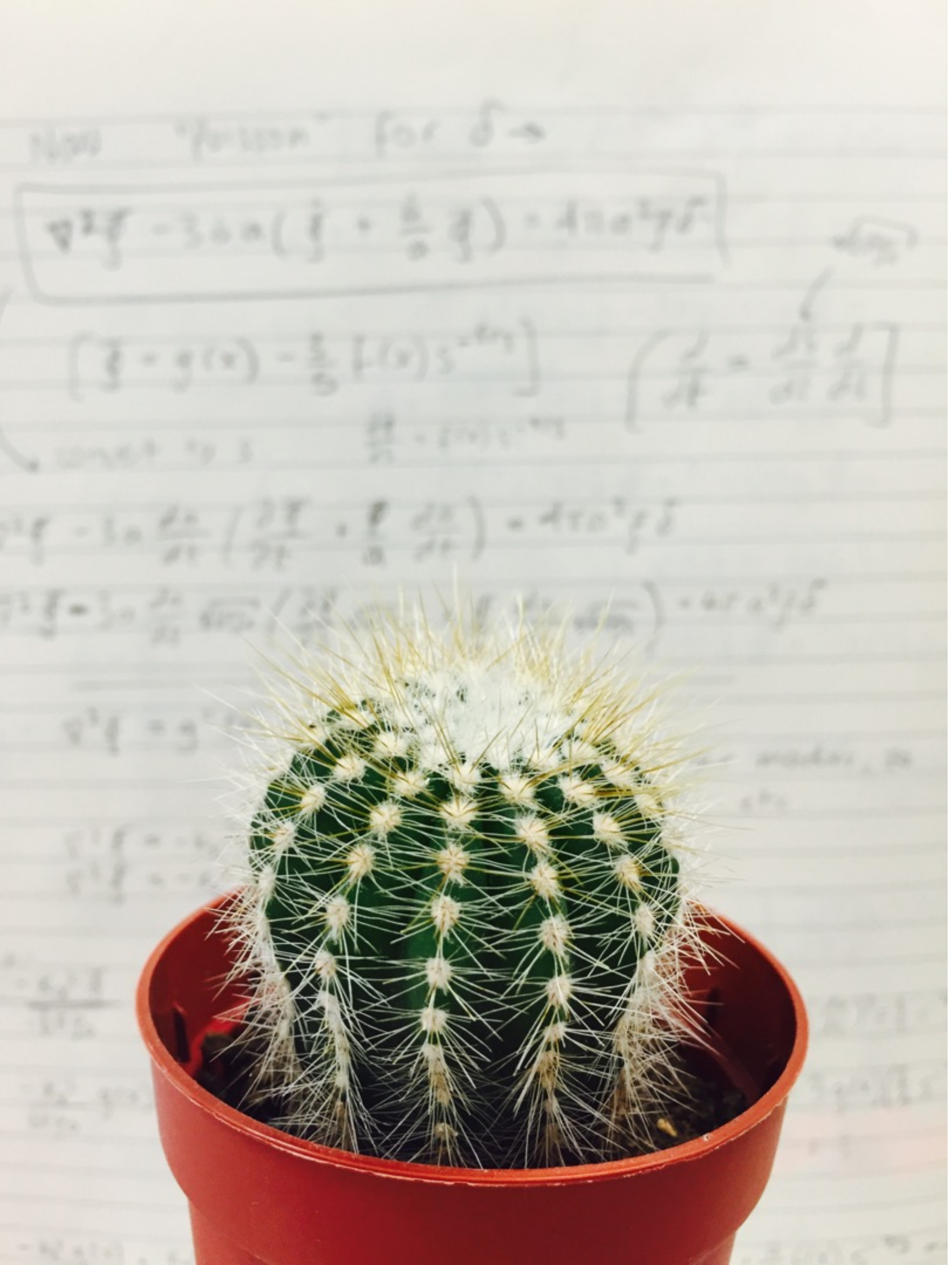


Image: David Liptai (Monash University)

Cactus

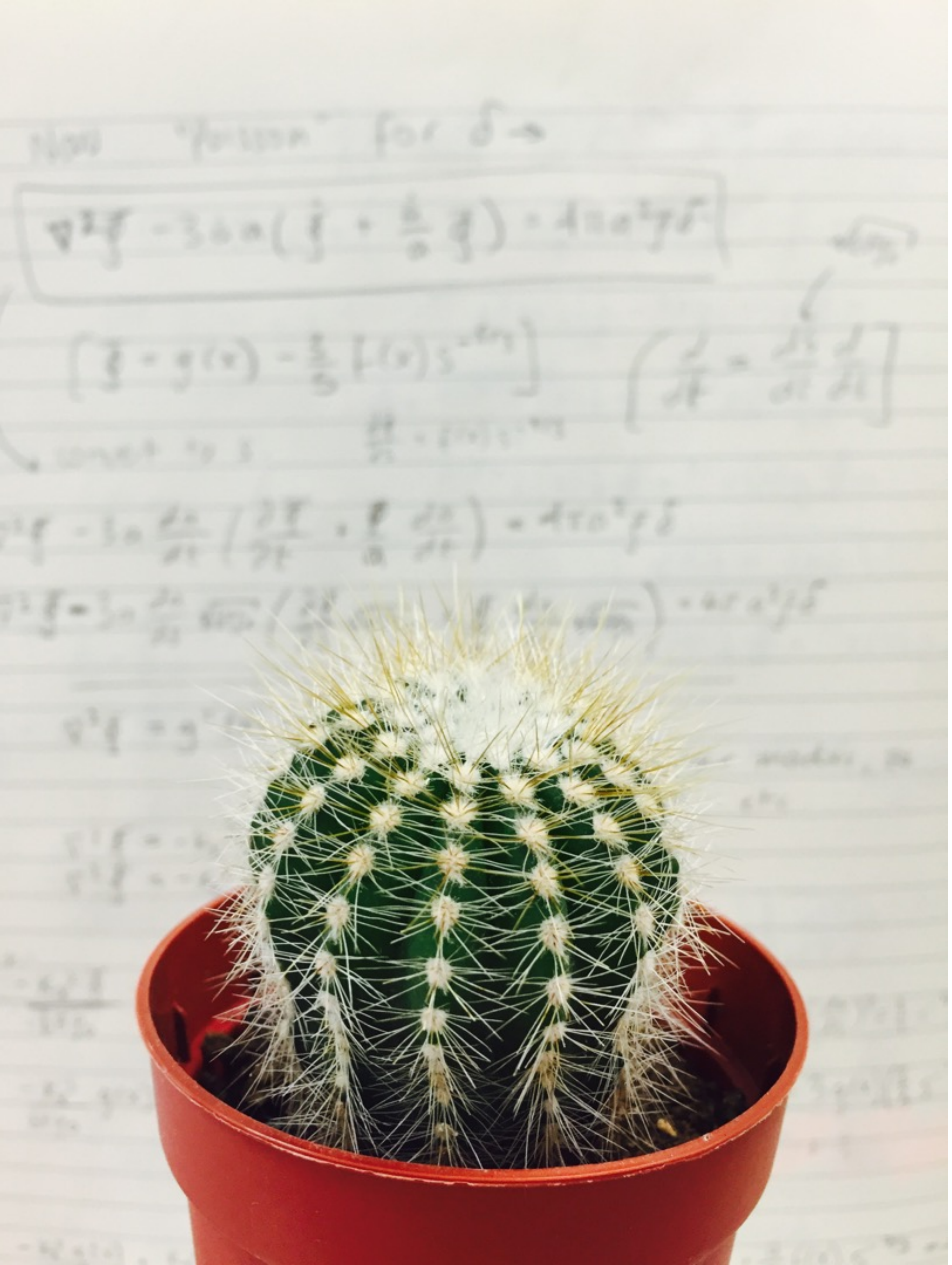
- Central core (flesh)
- Application modules (thorns)
- Einstein Toolkit
- Our thorn: **FLRWSolver**



Cactus

- **GRHydro**
  - Polytropic EOS: $P = K\rho^\gamma$
  - $K = 10^{-3}$
  - $\gamma = 2$
  - $P_{\rm init} \approx 10^{-19}$
  - Limitation, but we match dust solution to within 10^{-3}

Image: David Liptai (Monash University)



Cactus

- **FLRWSolver**
  - Initialises matter distribution

$$P \quad \rho \quad v^i$$

  - Initialises metric and extrinsic curvature

$$\gamma_{ij} \quad K_{ij} \quad \alpha \quad \dot{\alpha} \quad \beta^i$$

  - Option to add linear perturbations

Image: David Liptai (Monash University)

Perturbations in FLRWSolver

$$ds^2 = a^2(\eta)[-(1+2\Psi)d\eta^2 + (1-2\Phi)\delta_{ij}dx^i dx^j]$$

$$\downarrow$$

$$\bar{G}_{\mu\nu} + \delta G_{\mu\nu} = 8\pi\left(\bar{T}_{\mu\nu} + \delta T_{\mu\nu}\right)$$

Perturbations in FLRWSolver

$$\nabla^2\Phi - 3\frac{\dot{a}}{a}\left(\dot{\Phi} + \frac{\dot{a}}{a}\Psi\right) = 4\pi\bar{\rho}\,\delta a^2$$

$$\frac{\dot{a}}{a}\partial_i\Psi + \partial_i\dot{\Phi} = -4\pi\bar{\rho}\,a^2\delta_{ij}\delta v^j$$

$$\ddot{\Phi} + \frac{\dot{a}}{a}\left(\dot{\Psi} + 2\dot{\Phi}\right) = \frac{1}{2}\nabla^2(\Phi - \Psi)$$

$$\Phi - \Psi = 0$$

Perturbations in FLRWSolver

$$\nabla^2\Phi - 3\frac{\dot{a}}{a}\left(\dot{\Phi} + \frac{\dot{a}}{a}\Phi\right) = 4\pi\bar{\rho}\,\delta a^2$$

$$\frac{\dot{a}}{a}\partial_i\Phi + \partial_i\dot{\Phi} = -4\pi\bar{\rho}\,a^2\delta_{ij}\delta v^j$$

$$\ddot{\Phi} + 3\frac{\dot{a}}{a}\dot{\Phi} = 0$$

Solutions to these provide initial conditions

Perturbations in FLRWSolver

$$\Phi = f(x^i)$$

$$\delta = \frac{a_{\rm init}}{4\pi\rho^*}\,\xi^2\,\nabla^2\Phi - 2\Phi$$

$$\delta v^i = -\sqrt{\frac{a_{\rm init}}{6\pi\rho^*}}\,\xi\,\nabla^i\Phi$$

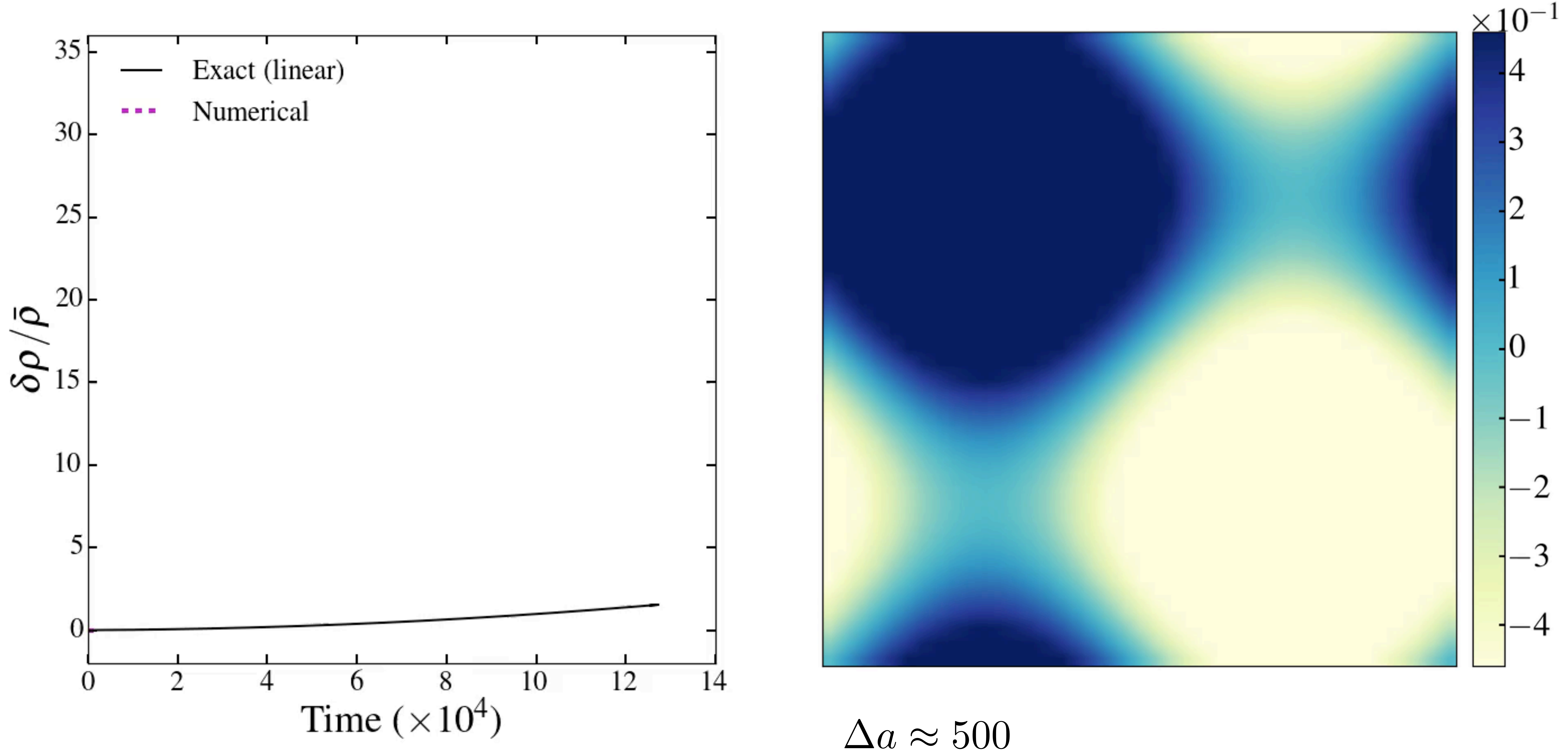
$$\xi \equiv 1 + \sqrt{\frac{2\pi\rho^*}{3\,a_{\rm init}}}\,\eta$$

Testing perturbations

$$\Phi = \Phi_0 \sum_{i=1}^{3} \sin\left(\frac{2\pi x^i}{L}\right)$$

$$\Phi_0 = 10^{-6}$$

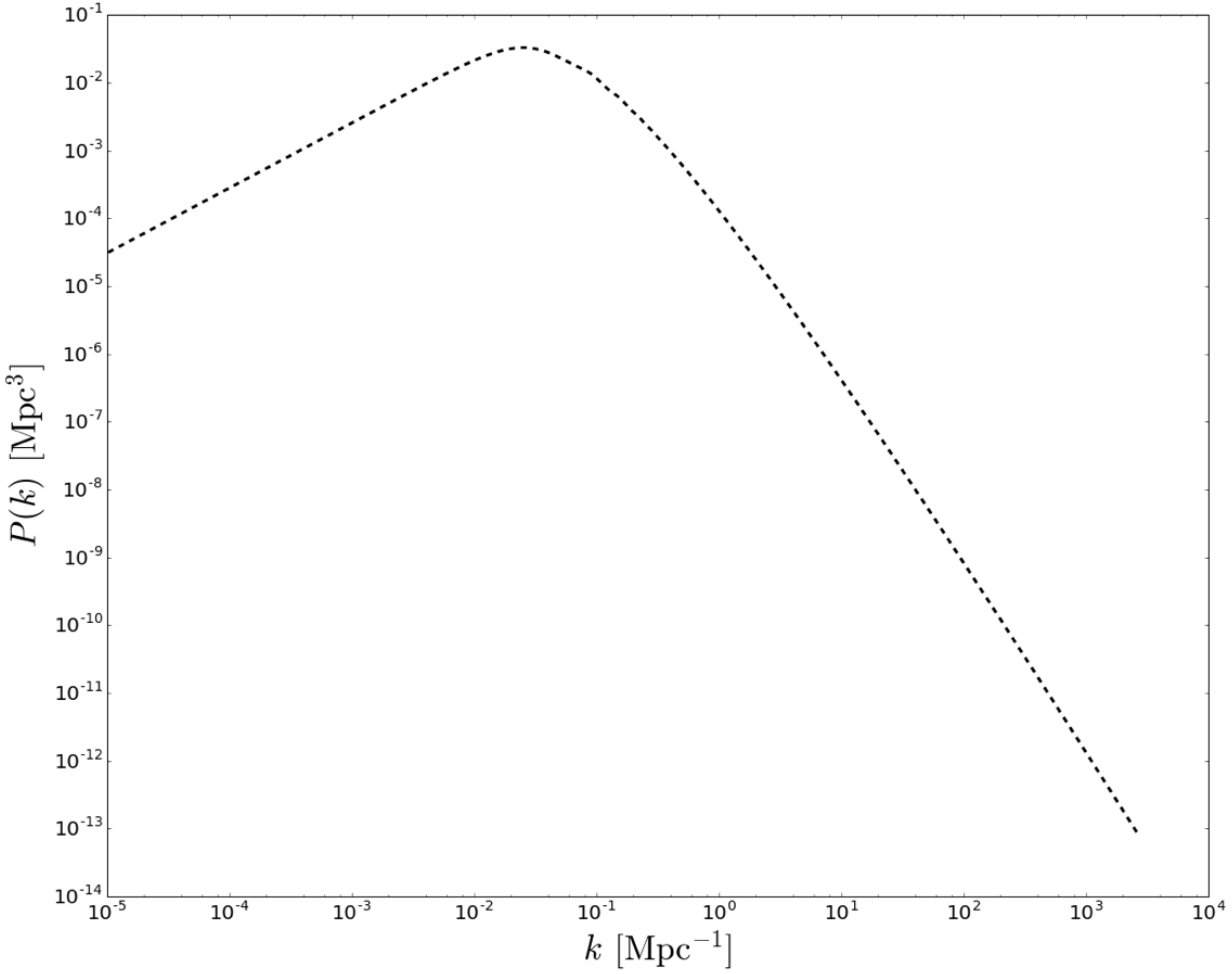
$$\Rightarrow \delta_0 \sim 10^{-3}, \quad \delta v_0 \sim 10^{-5}$$



$\Delta a \approx 500$

Macpherson et al. (2017)

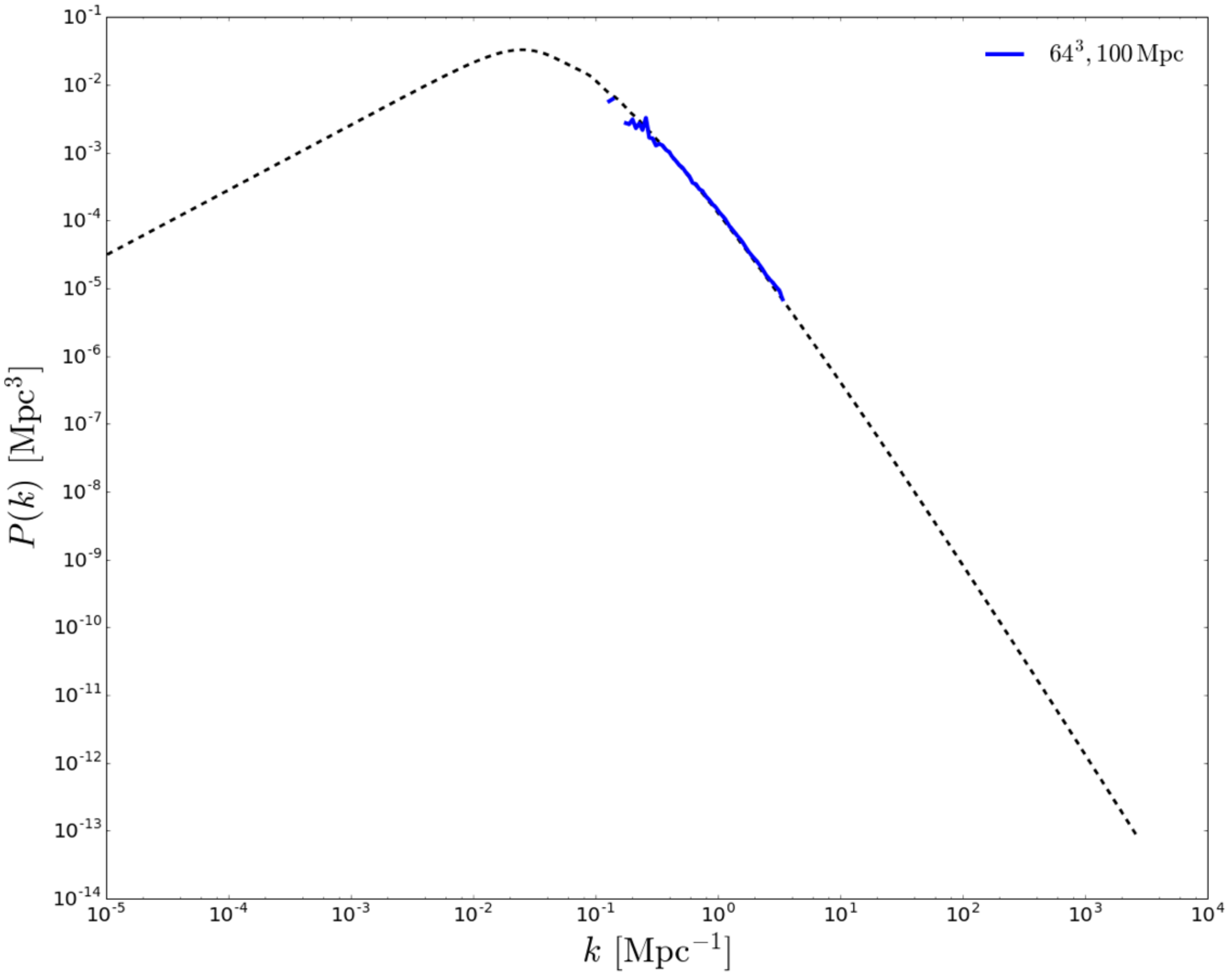
Initial conditions: a homemade CMB



- Code for Anisotropies in the Microwave Background (CAMB) + Planck (2015) data
- Matter power spectrum at CMB with $z = 1100$

Macpherson et al. (2017b; in prep)

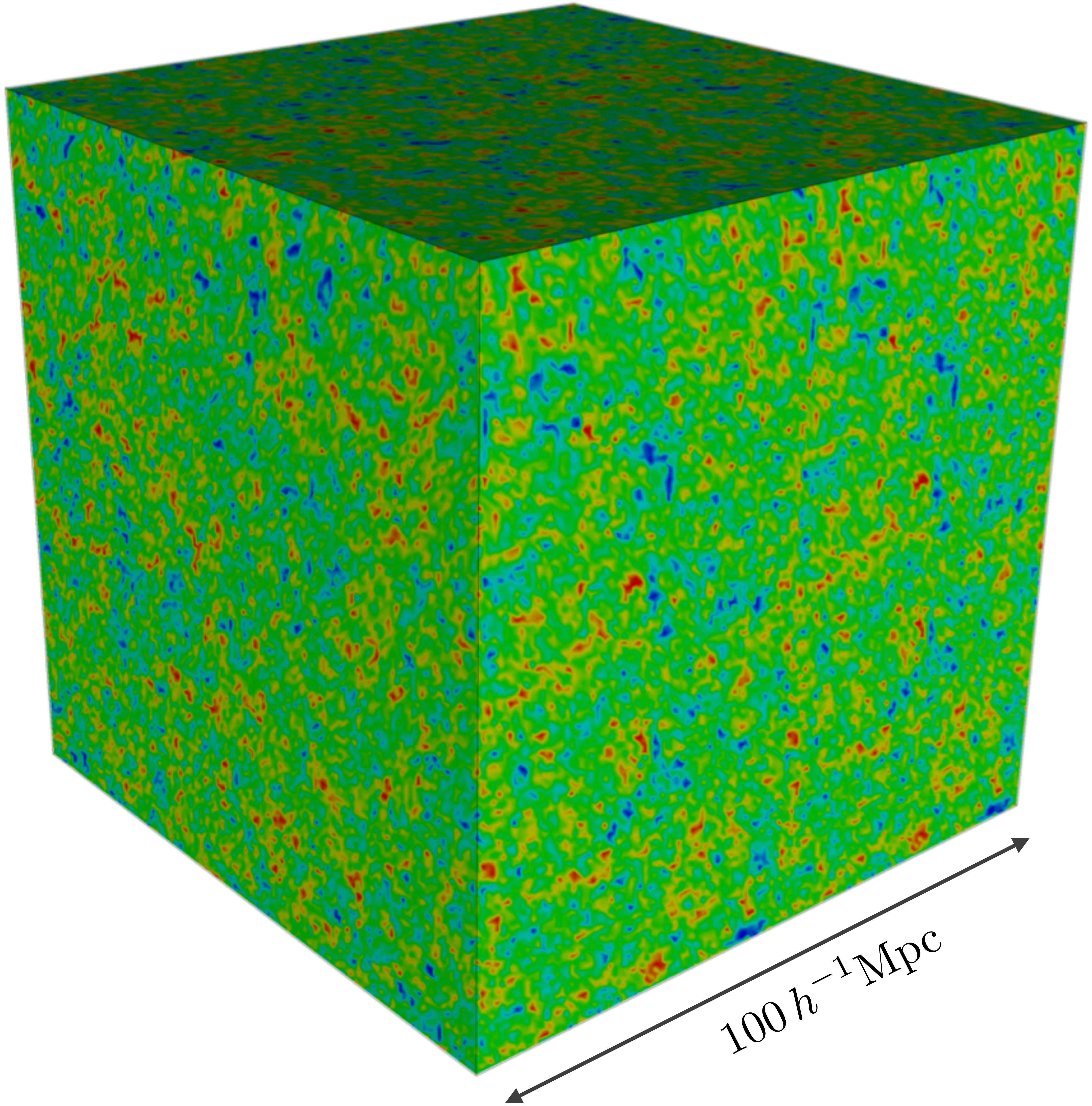
Initial conditions: a homemade CMB

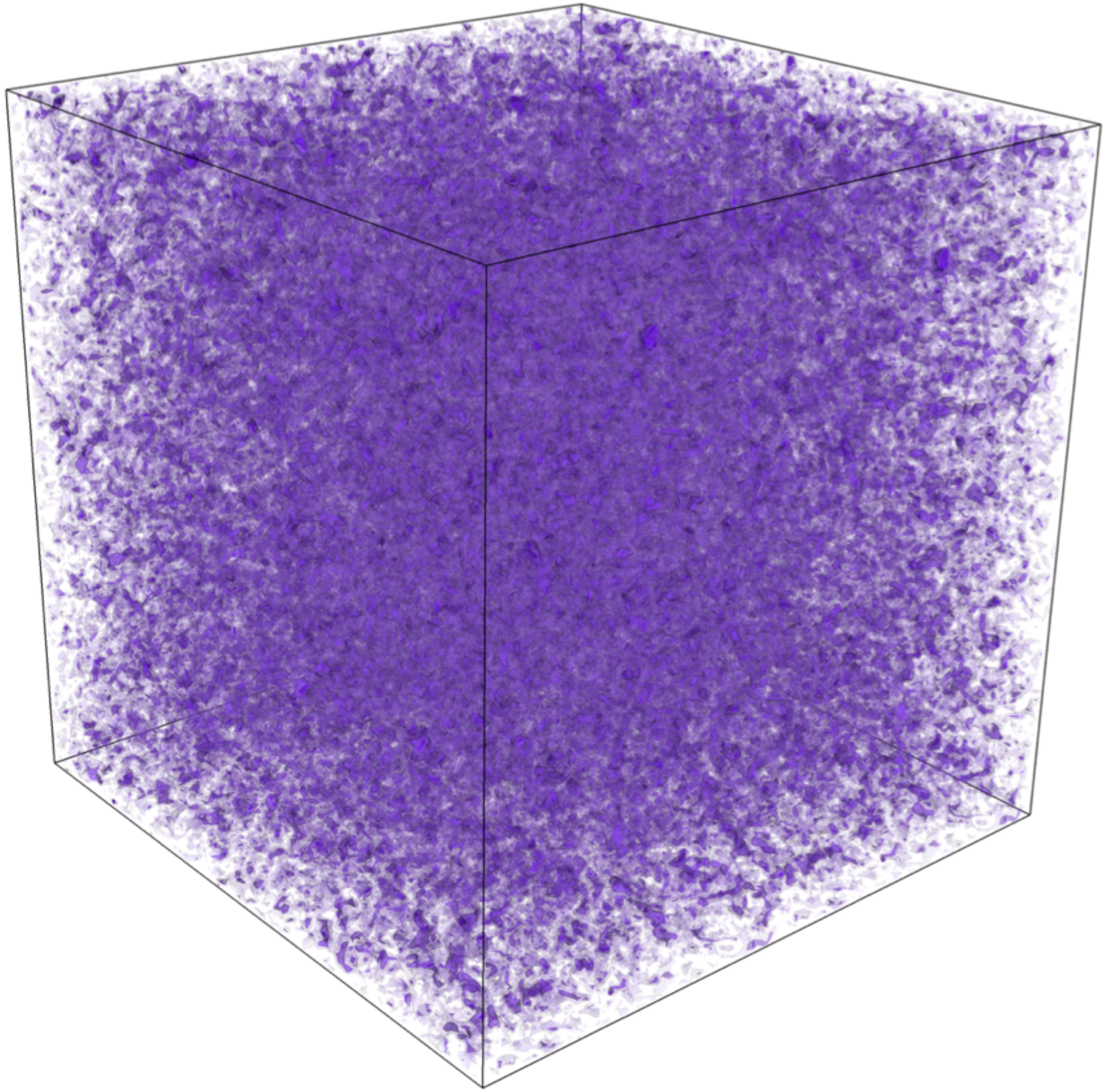


- Create gaussian random field ($\delta\rho/\bar{\rho}$)
- 64^3 domain with 100 Mpc on a side
- Sample P(k) up to Nyquist frequency: $\lambda_{\rm min} \sim 2\Delta x$
- Calculate Φ and δv^i from this field

Macpherson et al. (2017b; in prep)

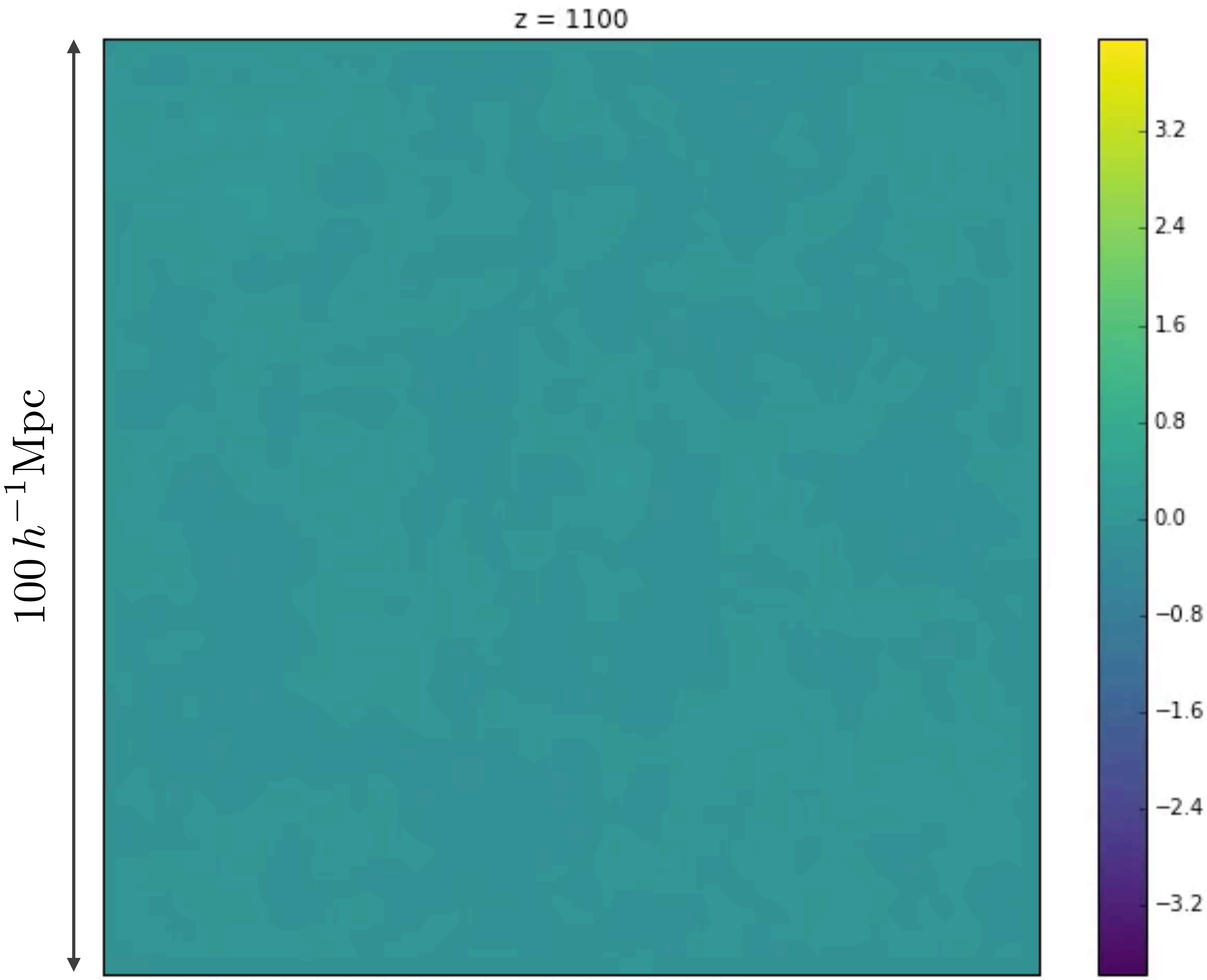
A homemade CMB



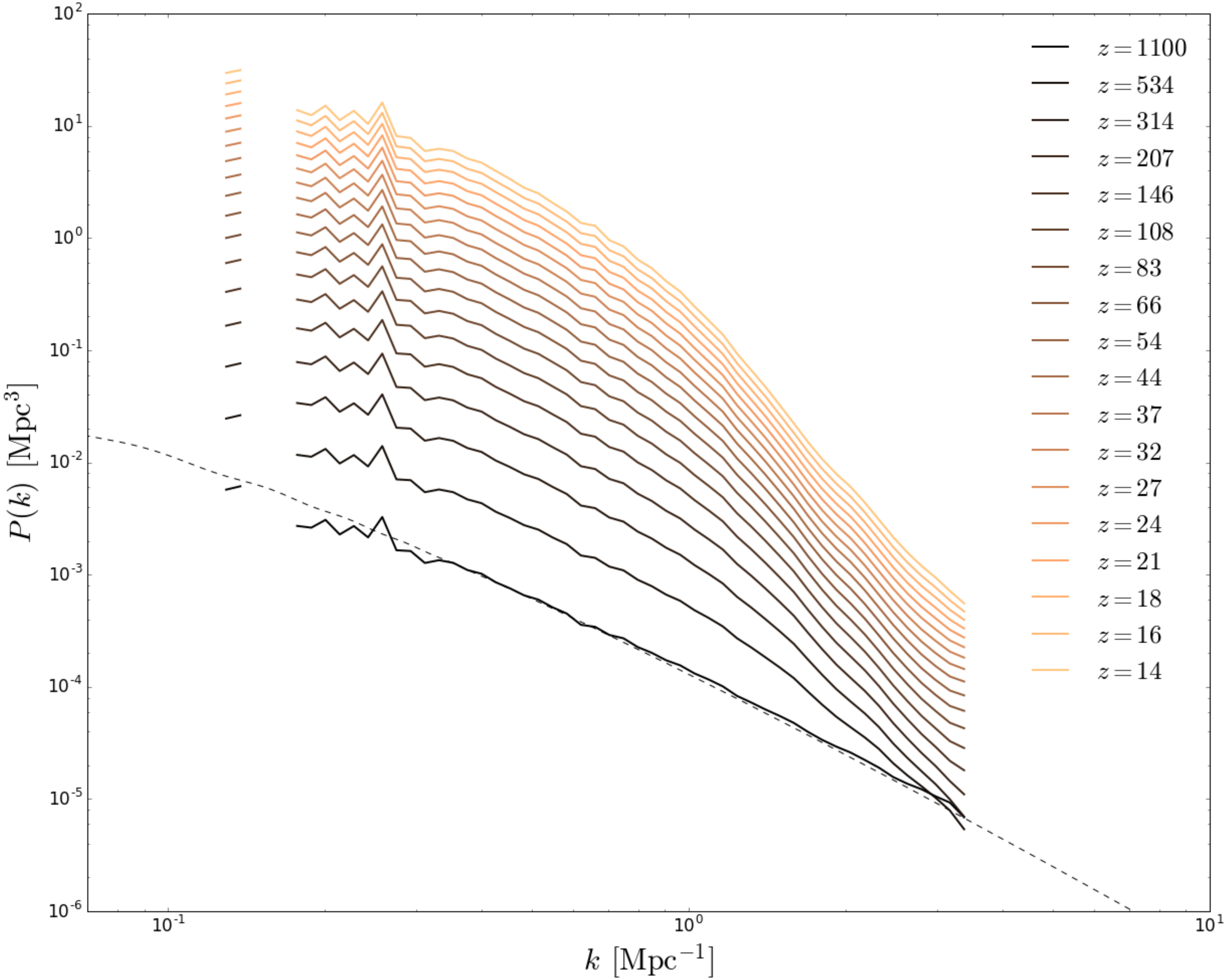


Macpherson et al. (2017b; in prep)

- 64^3 domain
- Sampling down to a few Mpc
- Ran through to $z \approx 1$ but no visualisation of this yet!
- Higher resolutions currently running…

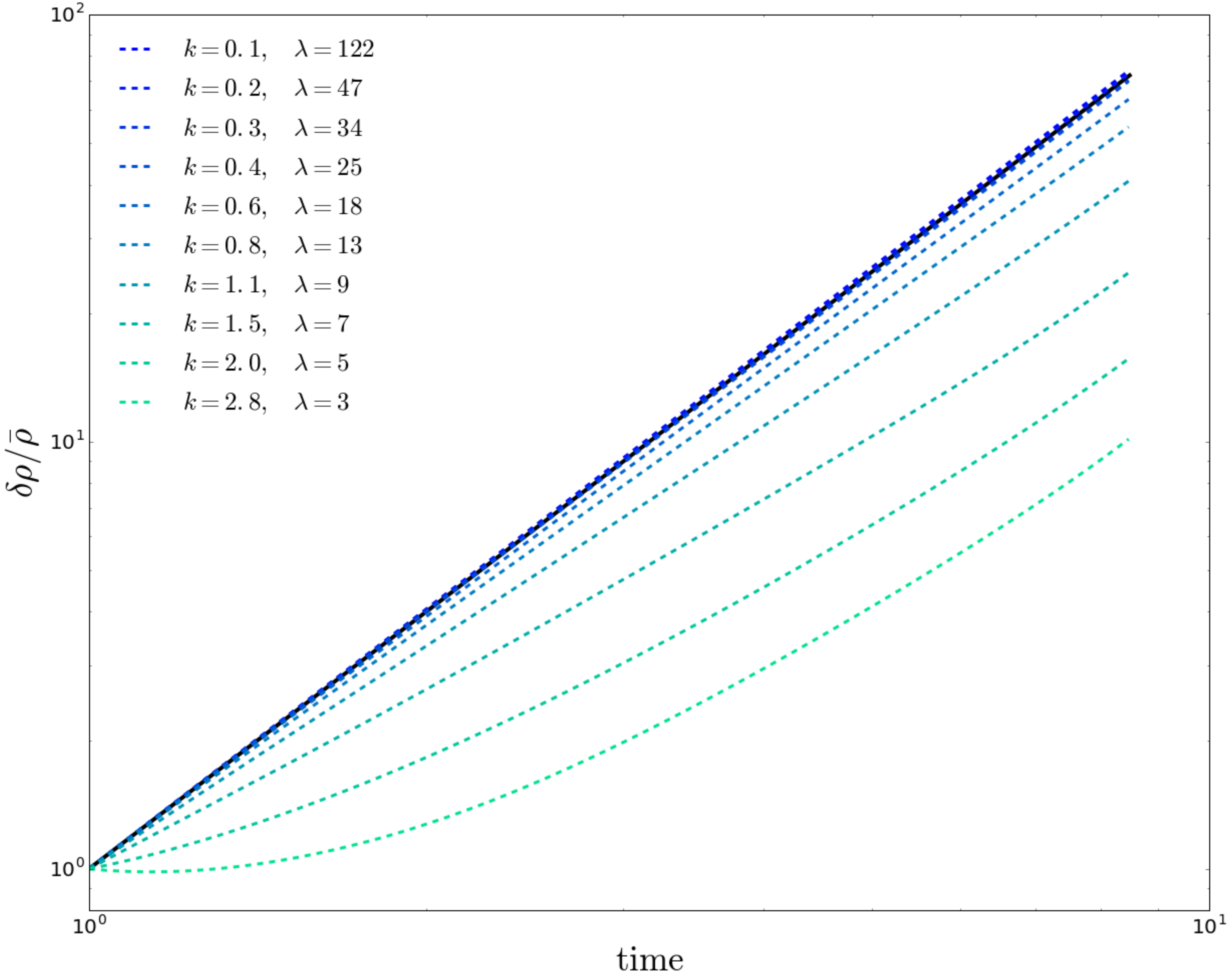


Macpherson et al. (2017b; in prep)



- 64^3, 100 Mpc box
- Tail of power spectrum damped
- Why?

Macpherson et al. (2017b; in prep)

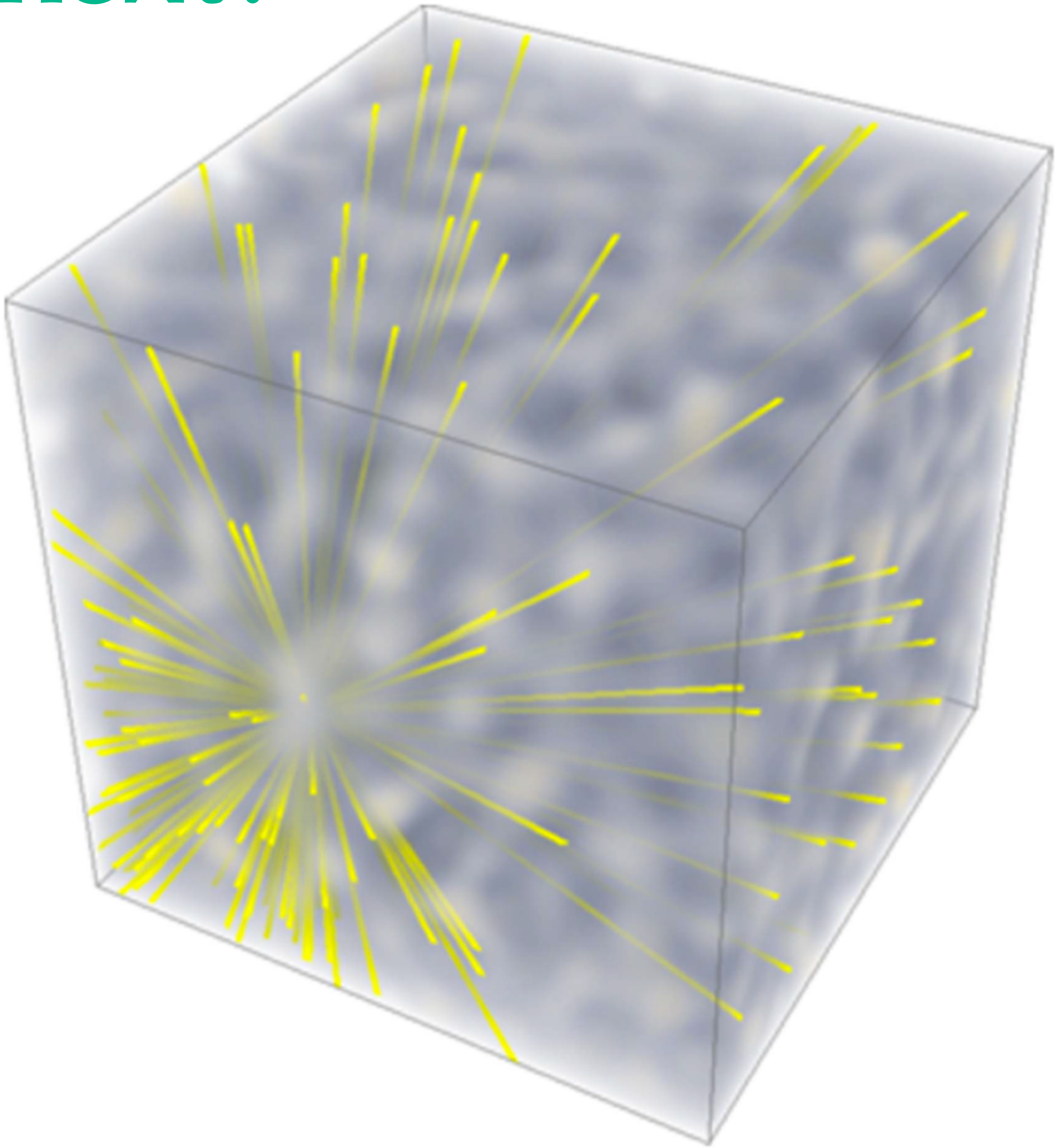


- 64^3, 100 Mpc box
- Damping at high frequencies: due to under-sampling?
- Do we cut these modes out?
- Testing in progress!

Macpherson et al. (2017b; in prep)

What's next?

- Synthetic observations:
  - Hubble diagrams (Giblin et al. 2016)
  - Integrated Sachs-Wolfe effect (structure and the CMB)
  - Velocity dispersions
  - Matter power spectrum at low redshift



Giblin et al. (2016)

Summary

- Precision of cosmological surveys is increasing
- Worthwhile to check full general relativistic effects
- We use Cactus & the Einstein Toolkit with **FLRWSolver**
  - Grown initially linear perturbations into the non-linear regime
  - Now simulating more realistic density distributions
  - Moving towards synthetic observations

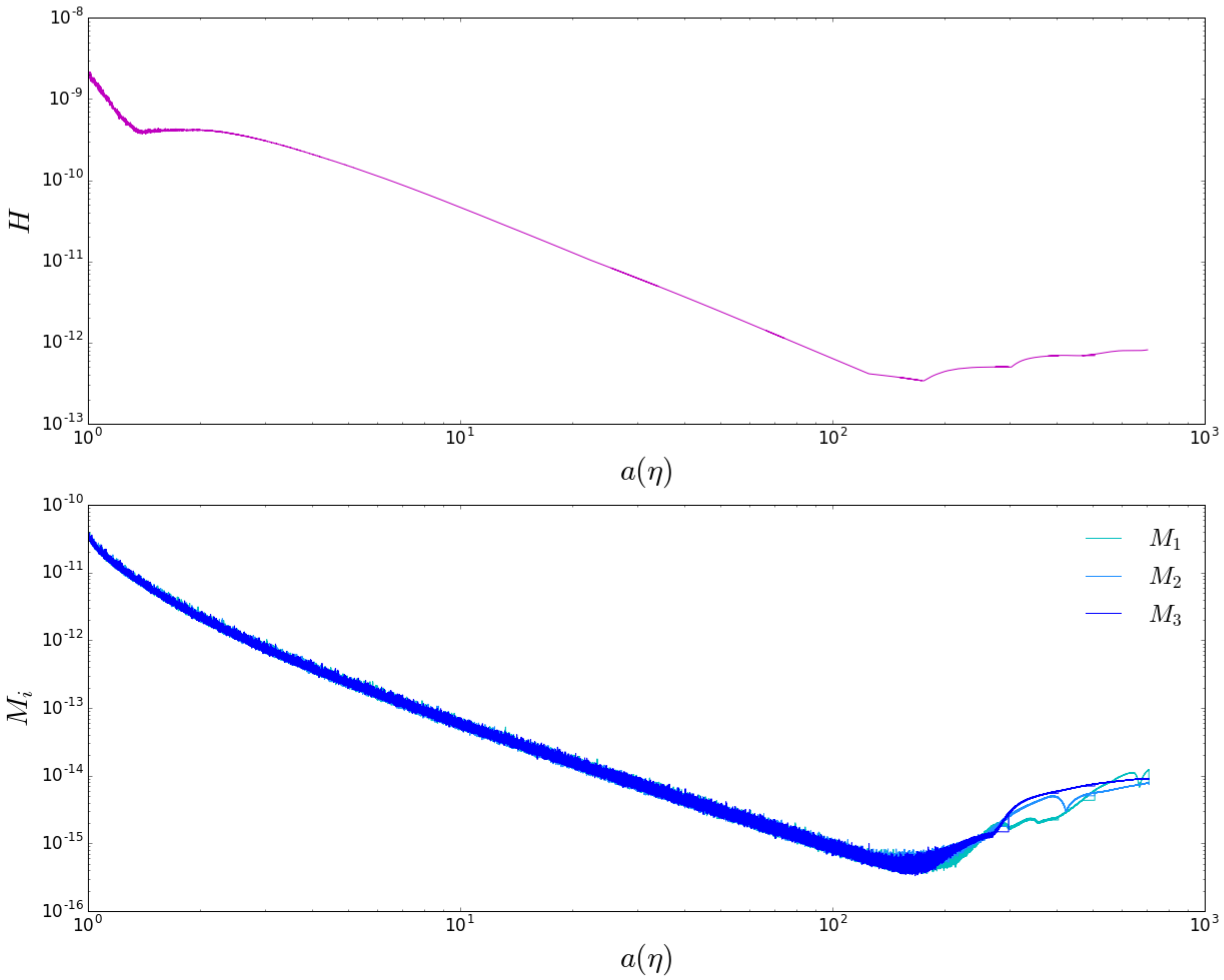
Constraints

Hamiltonian constraint

$$ {}^{(3)}R - K_{ij}K^{ij} + K^2 - 16\pi\rho = 0 $$

Momentum constraint

$$ \nabla_j K^j{}_i - \nabla_i K - \gamma_{ij}\,\rho\, u^j = 0 $$



H
a(η)
M_i
a(η)
M_1
M_2
M_3